

Stereometry;

Or The ART of

GAUGING

Made easie, by the Help of a

Sliding-Rule:

Which shews the Area's of Circles in Gallons, and the Square and Cube-Root of any Number under 100000, by Inspection: And is also very useful in *Arithmetick* and *Geometry*, particularly in the Measuring of *Superficies* and *Solids*.

WITH AN

Appendix of CONICK SECTIONS;
Shewing the *Nature*, *Properties* and *Contents* of
several *Solids*, frequently mentioned in Books of
GAUGING.

By THO. EVERARD, Esq;

The NINTH EDITION Carefully Corrected.

To which are Added, *New EXCISE-TABLES*:
Also the Method of Calculating *Diagonal Lines*;
with Short and Plain Rules for Gauging of
Malt; and the Line upon the RULE call'd
M. D. or the *Malt-Depth*, Explain'd.

LONDON, Printed for Mary Haffey in Little Britain, R. Wilkin, D. Midwinter, A. Bettefworth, B. Motte, and J. Lacy. MDCCLXXVII.

2. *Extremities*

GAUGING

Sliding-Knife:

[illegible]

W A H T I N - 3

Appendix of Conical Sections



LONDON: Printed for J. WILKINSON, at the
Printers, in Pall Mall.

TO THE
Right Honourable
SIDNEY
LORD Godolphin,
L A T E
Lord HIGH-TREASURER
O F
ENGLAND;

This T R A C T is Humbly
Dedicated and Presented
B Y T H E
AUTHOR.

®

TO THE

Right Honourable

SIR D. N. E. Y.

Lord Chancellor

LATE

Lord High-Treasurer

OF

ENGLAND;

This TRACT is Humbly

Dedicated and Presented

BY THE

AUTHOR.

A 2



THE

PREFACE.

THE Instrument describ'd in this Tract, (tho' useful to all Persons concern'd in the Measuring of Superficies and Solids) was principally design'd for the Assistance of the Officers of the Excise; for whom it is now found so useful and necessary, that no Gauger is thought to be well prepared for his Business without it: But the many Thousands that have been Sold, and the several Editions

The P R E F A C E.

of this Tract, are so good Evidence of its Utility, that it is as needless as unbecoming, for me to say any thing in its Commendation.

If the R E A D E R finds this Edition somewhat different from some of the former, it is presumed he will have no cause to complain of the Alterations; for besides, that this is more Portable, and more Correct, here is also added [Sect. IX.] some Short and Plain Rules for the Gauging of Malt; with a particular Account of the Dimensions and Content of the Standard-Bushel, and how the same were found and ascertained; also a Table shewing the Diameter and Depth of any legal Bu-

The P R E F A C E.

Bushel, Half-Bushel, Peck, or Gallon, with Directions for making any of these Measures, and for Rectifying such as are already made.

 In this Edition are added New Tables of Excise agreeing to the Act of Parliament made in the Year 1710, for laying Additional Duties upon all Exciseable Liquors, advancing the Excise Three pence per Barrel strong, and One penny per Barrel small Beer. In the said Tables I have kept the same Method as was observ'd by our Author, which were Calculated for the Common Brewery both in Town and Country; and I have added two Cash-Tables for Victualers both for strong and small Beer, which were never in

THE PREFACE.

any of the Former Editions, And that this Book and Sliding-Rule may be Compleat Companions for the Officers of the Excise, &c. I have Enlarg'd the short Rules for Gauging of Malt, and explain'd the Gauge-Point, and the Lines marked MD, or Line called the Malt-Depth, shewing its great use in casting up Malt-Gauges.

The present Proprietors, being desirous that this Edition should as far as possible appear perfect, have requested me to Revise the following Sheets, and to add or alter what I thought was most necessary, which I have accordingly done, and that not Transiently, but with care have wrought every Operation over a
new

THE PREFACE.

new, both by Pen and Sliding-Rule ;
so that I hope very few (if any)
Errors have escaped.

To my knowledge this Tract and
Sliding-Rule are so well known a-
mongst the Officers of his Majesty's
Revenues, that there is no room to
doubt, but that the said Book and
Rule, will be in great Esteem as
long as the Excise it self shall
continue.

And here further let me acquaint
the Reader, that the Line E, is not
only useful to Extract the Cube-
Root, and to find the Solidity of a
Sphere, as mentioned in Pages 36,
and 62, but is also more particularly
useful to all concern'd in Stereometry,
for calculating Diagonal Lines to any
Cask whatsoever, as I shall here
shew, viz. As

The PREFACE.

As the known Length of any *Diagonal Line* on the Line *D*, in Inches,
Is to the Content of the same Cask
on the Line *E*, in Gallons,
So is the Content of any other Cask
in Gallons, on the Line *E*.
To the *Diagonal* of that Cask in In-
ches on the Line *D*.

Example. Admit a Cask in the form
of a *Spheroid*,

whose	{	Head—22.	}	Inches.
		Bung—25.		
		Length 30.		
		Diagonal 27.8		
		Content 48.3		
			Ale-Gallons.	

O P E R A T I O N.

Set 27.8 on *D*, to 48.3 on *E*, and as
the Rule now stands, it is a Table: So
against any Number of Gallons on the
Line *E*, you have given on the Line *D*,
the

The P R E F A C E

the Length of the *Diagonal* in Inches and Decimal Parts. Thus,

Content	10.	20.	30.	40.
Diagonal	16.44	20.8	23.8	26.25

For WINE.

Let the Dimensions of a Spheroidal Cask be as above, the Content in Wine-Gallons will be 58.97; then for the *Diagonal*, set 27.8 on *D*, to the Content 58.97 on *E*, and as the Rule now stands, you have the *Wine-Diagonal* for a *Spheroidal Cask*. Thus,

Content	10.	20.	30.	40.
Diagonal	15.32	19.32	22.1	24.36

And so on *ad infinitum*.

C. LEADBETTER.

Adver-

THE PRESS
AND BOOKS
OF THE
ADVERTISEMENT.

THIS SLIDING-RULE, and all sorts
of Instruments useful in Gauging,
Surveying, Dialling and all Practical Parts
of the Mathematicks; are made to the
greatest Exactness, and Sold at the most
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THE
INTRODUCTION
OF
Decimal Arithmetick.

WHOEVER will rightly understand the Art of *Gauging*, or the *Use* of the *Instrument* here described, ought in some measure to be acquainted with the Art of *Arithmetick*; at least with *Numeration*, *Addition*, *Substraction*, *Multiplication*, and *Division*, both in *Whole Numbers* and *Decimal Fractions*.

And forasmuch as most of the *Problems* in this *Traſt* are reſolved by help of the *Line of Numbers*, the ſeveral *Uſes* of which *Line* are render'd more eaſie by the Knowledge of *Decimal Arithmetick*; I ſhall in the firſt place endeavour to ſhew what a *Decimal Fraction* is, and alſo give ſome *Rules* and *Ex-amples* by which any Perſon who does at all underſtand the *Vulgar Arithmetick*, may in a few Hours time thoroughly comprehend this.

A *Decimal Fraction* is that, which by prefixing a Point or a Prick towards the left Hand, its Value is decreas'd from ſo many Units, to ſo many tenth Parts of an Unit; and if a Point and a Cypher, or Digit be prefix'd, it will then be ſo many hundred

B Parts

parts ; and if a Point and two Cyphers or Digits be prefixed, its value is decreas'd to be but so many thousand parts : As, if you would prefix before the Figure 2, a Point thus [.2] 'tis then decreas'd from 2 Units, to 2 tenth parts of an Unit. For,

As in *Whole Numbers* the Value or Denomination of places *increases* by Tens from the Unit's place towards the *left hand* ; so in *Decimal Fractions*, the Value or Denomination of Places *decreases* by Tens from the Unit's place towards the *right hand* ; as in the following Scheme ;

In which I have set an Unit in its due place, and all the Figures towards the left hand (being *Whole Numbers*) do *increase* by Tens ; for the Figure 2, next the Unit's place is 2 Tens (or 20,) the next is 3 Hundreds, and the next 4 Thousands, &c.

Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundred Parts.	Thousand Parts.	Ten Thousand Parts.	Hundred Thousand Parts.
6	5	4	3	2	1	.2	3	4	5	6

But all the Figures from the Unit's place, towards the right hand (being *Decimal Fractions*) do *decrease* by Tens ; so the Figure 2, towards the right hand is 2 tenth parts, the next is 3 hundred parts, and the next 4 thousand parts of an Unit ; each place towards the right hand being ten times less. In

Decimal Arithmetick.

3

In *Decimal Arithmetick*, we alway imagine (and it would be very commodious if it were really so) that all intire Units, Integers, and Things, are divided into ten equal Parts, and each of these Parts so divided we call *Primes*; we also divide each of these *Primes* into other ten equal Parts, and every of these Divisions we call *Seconds*; and each of these being divided into ten other equal Parts, may be called *Thirds*, (and as one *Prime* is $\frac{1}{10}$ of an Unit, so one *Second* is $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$ of an Unit; and one *Third* is $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{100}$ of $\frac{1}{10}$ Part of an Unit) so by dividing the former, and subdividing these latter, we may run on *ad infinitum*.

Let a Pound Sterling be given to be Decimally divided.

According to the Notion premised, the first Division must be *Primes*, the next Division *Seconds*, the next *Thirds*, &c.

So if one Pound Sterling (being 20*s*.) be divided into ten equal parts, one of these Parts is one *Prime*, and its Value is 2*s*. and will stand thus [.1] three *Primes*, or three *tenths* of a Pound will stand thus [.3] and its Value is 6*s*. Again, one *Prime*, or .1 being divided into ten equal parts, each of those Parts will be one *Second*, or one hundredth part of a Pound, and is thus express'd [.01] and its Value will be 2*d*. 1*f*. and $\frac{6}{10}$ of a Farthing; and if .01 be divided into ten other equal parts, each of those parts will be *Thirds*, or one thousandth part of a Pound, and will stand thus, [.001] and its Value will be $\frac{2}{100}$ of a Farthing.

The like may be understood of one Pound *Troy*, or *Averdupois*, one Foot, one Gallon, or any other Integer, or Thing whatsoever.

A *Decimal Fraction*, whether it stand alone, or be join'd with *Whole Numbers*, is distinguish'd from a *Whole Number* by a Point or Prick before it, towards the left hand, as in these Examples.

.4752 4.752 47.52 475.2

In *Decimal Fractions* the *Numerators* only are set down, the *Denominators* being known by the Number of Places in the *Numerator*: For if the *Numerator* consist of but one Place, the *Denominator* is 10; if of 2 Places, the *Denominator* is 100, &c.

EXAMPLE.

The Denominator of $\left. \begin{array}{l} .2 \\ .25 \\ .051 \\ .0752 \end{array} \right\}$ is $\left\{ \begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \end{array} \right\}$

As Cyphers before a *Whole Number* have no Value, so Cyphers after a *Decimal Fraction* are of no signification, and therefore will not increase the Fraction, nor alter the *Denominator*: For .2 is two tenth Parts, and .20 is no more.

Again, as Cyphers after a *Whole Number* do increase that Number, so Cyphers before a *Decimal Fraction* do diminish the Value thereof; as in these Examples,

$\left. \begin{array}{l} .5 \\ .05 \\ .005 \\ .0005 \end{array} \right\}$ is 5 $\left\{ \begin{array}{l} \text{Tenth} \\ \text{Hundred} \\ \text{Thousand} \\ \text{Ten thousand} \end{array} \right\}$ Parts.

Here

Here every Cypher added, removes the Fraction further from Unity, making it ten times less than before.

Addition and Substraction of Decimal Fractions.

1. **A**S for the Operation of *Addition and Substraction* in *Decimals*, it is the *very same* with the *Vulgar*; there must only care be had of placing *Units* under *Units*, and *Fractions* under *Fractions* in their proper Ranks and Files.

For Example.

To _____ 376.435
 Add _____ 54.22
 Sum is _____ 430.655

Again,

To _____ 26.51
 Add _____ 527.342
 Sum is _____ 553.852

Subtraction.

$$\begin{array}{r}
 \text{From} \quad \text{—————} \quad 430.645 \\
 \text{Subtract} \quad \text{—————} \quad 54.210 \\
 \hline
 \text{Remains} \quad \text{—————} \quad 376.435
 \end{array}$$

Or, thus :

$$\begin{array}{r}
 \text{From} \quad \text{—————} \quad 553.852 \\
 \text{Subtract} \quad \text{—————} \quad 527.342 \\
 \hline
 \text{Remains} \quad \text{—————} \quad 26.510
 \end{array}$$

Multiplication of Decimals.

1. **I**N *Multiplication* both of *Decimal Fractions* and *Mixt Numbers*, there are ever *two* Numbers given to find a *third* unknown.

One of the Numbers given (and it's no matter which) is called the *Multiplicator*, the other the *Multiplicand*, and the Number sought is call'd the *Product*; and this doth ever contain one of the Numbers given, as many times as the other given Number contains Unity.

Multiplication of Decimals is performed after the *same manner* as in *Whole Numbers*; for the Numbers being set down one under another, we proceed in the *Multiplication* as if they were all *Whole Numbers*; only it must be observ'd (when the Operation is finish'd) how *many* Places of *Decimal Fractions* are contain'd in the *Multiplicator* and *Multiplicand*; for *so many* must be in the *Product*, as in these Examples.

Mul-

$$\begin{array}{r}
 \text{Multiplicand} \quad \text{---} \quad 35.25 \\
 \text{Multiplier} \quad \text{---} \quad 7.24 \\
 \hline
 14100 \\
 7050 \\
 24675 \\
 \hline
 \text{Product} \quad \text{---} \quad 255.2100
 \end{array}$$

Or, thus :

$$\begin{array}{r}
 \text{Multiplicand} \quad \text{---} \quad 6.75 \\
 \text{Multiplier} \quad \text{---} \quad .25 \\
 \hline
 3375 \\
 1350 \\
 \hline
 \text{Product} \quad \text{---} \quad 1.6875
 \end{array}$$

2. When it happens (as it often does) that there are not so many *Figures* in the *Product*, as there are *Fractions* in the two Numbers to be multiply'd, you are then to place *Cyphers* before the *Product*, till the Number of Places be equal ; as in these Examples.

$ \begin{array}{r} .04 \\ .6 \\ \hline .024 \end{array} $	$ \begin{array}{r} .75 \\ .05 \\ \hline .0375 \end{array} $	$ \begin{array}{r} .85 \\ .005 \\ \hline .00425 \end{array} $
--	--	--

3. As *Whole Numbers* are increased by Multiplication, so *Decimal Fractions* are hereby made less ; for the *Product* is remov'd farther from Unity than either of the *Fractions* given to be multiply'd ; as appears by the last Examples.

B 4

4. When

4. When a *Decimal Fraction* or *Mixt Number* is to be multiply'd by an Unit with Cyphers (as 10, 100, 1000, &c.) you need only remove the Point so many Places towards the right hand, as there are Cyphers with the Unit : Thus, if .7562 be multiply'd

$$\text{By } \left\{ \begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \end{array} \right\} \text{The Product will be } \left\{ \begin{array}{l} 7.562 \\ 75.62 \\ 756.2 \\ 7562. \end{array} \right\}$$

Division of Decimals.

1. **I**N *Division* there are Two Numbers given, to find a Third unknown.

One of the Numbers given is call'd the *Divisor*, the other the *Dividend*, and the Number sought is call'd the *Quotient*, and this *Quotient* doth ever contain Unity as many times as the *Divisor* is contain'd in the *Dividend*.

Division of Decimal Fractions, or *Mixt Numbers*, is perform'd after the same manner as in *Whole Numbers*; all the Difficulty is to know the Denomination of Places in the *Quotient*, and they are easily discover'd by this one General Rule.

The first Figure in the Quotient must be of the same Denomination with that Figure in the Dividend, which at the first demand stands exactly over the Place of Unity in the
Divi-

Decimal Arithmetick.

9

Divisor. I shall make this plain by some Examples.

Example.

Let it be requir'd to divide 32 by 51.2.

In this Example the *Divisor* (51.2) is greater than (32) the *Dividend*; therefore in this and the like cases you must place a competent Number of *Cyphers* behind the *Dividend*, which (if it be a Whole Number) the *Cyphers* so placed must be distinguish'd there-from as *Fractions*; as, here I put 4 *Cyphers* behind 32; and placing the Numbers in this manner,

(51.2) 32.0000

I proceed to the Division (as if all were Whole Numbers,) and first ask, how many times the *Divisor* is contained in (320) the three first Figures of the *Dividend*? And seeing in this Example it is not contain'd in

51.2) 32.0000 (6
128

the three first Figures, it will therefore extend to the fourth place, over which I put a Prick, to shew that I must begin to subtract at that place. I then ask, how many times 5 in 32? I find 6; I place 6 in the *Quotient*, and multiply the *Divisor* by 6, subtracting the *Product* out of the *Dividend*, beginning my Subtraction at the Place over which I put the Prick: Thus, 6 times 2 is 12, 12 from 0 I cannot, I borrow 2, and say, 12 from 20, and there remains 8, which I place under the *Cypher*: Again, 6 times 1 is 6, and 1 I borrowed is 7; 7 from 10, and there

B 5

there rests 2, which I place under the next Cypher before 8, as in the Example; then 6 times 5 is 30, and 1 I borrowed is 31; 31 from 32, and there rests 1, which I set under 2; then for my new Dividend I bring down the next Cypher, and set it behind the Remainder, and the Example will stand thus:

$$\begin{array}{r} 51.2 \overline{) 32.0000} \quad (6 \\ 1280 \end{array}$$

Then proceeding in my Division, I ask how many times 5 in 12; I find 2, which I place in the *Quotient*, and say, 2 times 2 is 4; 4 from 10, and there remains 6, which I set under 0: Again, 2 times 1 is 2, and 1 I borrowed, is 3; 3 from 8, and there rests 5; which I place under 8; then 2 times 5 is 10; 10 from 12, and there rests 2; which I place under 2; and to this Remainder I bring down another Cypher to make my new *Dividend*; so will the Example stand thus:

$$\begin{array}{r} 51.2 \overline{) 32.0000} \quad (62 \\ 1280 \\ 2560 \end{array}$$

This done, I enquire how many times 5 in 25, and find 5; which I place in the *Quotient*, and say, 5 times 2 is 10, 10 from 10, and there rests 0; then 5 times 1 is 5, and 1 I borrowed is 6, 6 from 6, and there rests 0: Lastly, 5 times 5 is 25, 25 from 25, and there remains 0; And thus my

Decimal Arithmetick.

II

my Division is finish'd, and the Example will stand thus :

$$\begin{array}{r} 51.2 \overline{) 32.0000} \quad (.625 \\ 1280 \\ 2560 \\ 0000 \end{array}$$

Now by the general Rule above given, it is plain that the first Figure in the *Quotient* must be Tenths, and therefore the Quotient in all Decimal parts ; for the *Divisor* being placed under the first Figures of the *Dividend*, this Example will stand thus :

$$\begin{array}{r} 32.0000 \quad (.625 \\ 51.2 \end{array}$$

By which it appears, that the *Cypher* in the *Dividend*, which stands over the *Unit's place* in the *Divisor*, is the place of *Tenths*.

See more Examples.

<i>Divisor.</i>	<i>Dividend.</i>	<i>Quotient.</i>
51.2)	32.00000	(.625
51.2)	32.00000	(.625
7.5)	.375	(.05

2. In Division of *Whole Numbers* or *Mixt*, if there happen to be a Remainder, you may bring down more *Cyphers*, and by continuing your *Division*, carry on the *Quotient* to as many places of *Fractions* as you please ; four or five will be very near the truth, but two or three will be sufficient in most Cases.

Ex-

Example.

$$\begin{array}{r}
 6.26) 25.800000 \quad (4.1214 \\
 \underline{760} \\
 1340 \\
 \underline{880} \\
 2540 \\
 \underline{36}
 \end{array}$$

3. When any Number (either *Decimal* or *Mixt*) is to be divided by an Unit with Cyphers, as 10, 100, 1000, &c. you need only remove the Prick in the *Dividend* so many Places toward the left Hand, as there are *Cyphers* with the Unit, supplying the vacant Places (if any be) with Cyphers.

Thus 756.2 divided by 10, is 75.62; divided by 100, it will be 7.562; by 1000, .7562; and if by 10000, the *Quotient* will be .07562.

4. *Divison* is prov'd by *Multiplication*, and *Multiplication* by *Divison*.

To prove *Divison*, multiply the *Quotient* by the *Divisor*, and the *Product* is the *Dividend*: Thus, .652 (the *Quotient* in the first Example of *Divison*) being multiplied by 51.2 (the *Divisor*,) the *Product* is 32.0000 (the *Dividend*.)

To prove *Multiplication*, divide the *Product* by either of the Numbers given to be multiply'd, and the *Quotient* will be the other Number given.

Thus, if the last *Product* (*viz.*) 32.0000, be divided by 51.2, the *Quotient* is .625; or if 32.000 be divided by .625, the *Quotient* will be 51.2.

And hence 'tis evident, that *Divison* is also proved by *Divison*: For if any *Quotient* be made a *Divisor*, it will quote the first *Divisor*. Example :

$$1.8) 28.8 \quad (16.$$

$$108$$

$$000$$

$$16.) 28.8 \quad (1.8$$

$$128$$

$$000$$

5. Now forasmuch as *Multiplication* is easier than *Division*, I shall here shew, that what is perform'd by a *Divisor*, may also be perform'd by a *Multiplicator*, and how such *Multiplicator* may be found: Suppose I were to divide 900 by 25, the *Quotient* will be 36.

Now let it be requir'd to multiply 900 by a certain Number that shall produce 36.

To find this *Multiplicator*, divide an *Unit* with Cyphers by 25, the *Divisor* propos'd, the *Quotient* will be .04, the *Multiplicator* sought: For 900 being multiply'd by .04, the *Product* will be 36, which was requir'd.

Again, Having a *Multiplicator*, to find a *Divisor*; this is but the *Converse* of the former: For if an *Unit* with Cyphers be divided by .04 (a *Multiplicator*) the *Quotient* will be 25, the *Divisor* as before.

6. A *Vulgar Fraction* is reduced into a *Decimal* by *Division*: For which the Rule is,

Divide the *Numerator* (of the *Vulgar Fraction* given) by the *Denominator*, and the *Quotient* will be a *Decimal Fraction*, equal in Value to the *Vulgar Fraction* given: So $\frac{1}{4}$ reduc'd as aforesaid, will be .25. *Example.*

$$4) 3.00 \quad (.75$$

$$20$$

$$00$$

Note,

Note, The *odd Parts* of an Integer cannot be exactly reduc'd to a *Decimal*, for there will always be a *Remainder*; in such Cases carry on the *Fraction* to four or five places, it will be very near the *Truth*. *Example*;

$$\begin{array}{r} 4 \\ 9 \end{array} \quad 9) 4.0000 (.4444$$

40

40

40

7. To reduce a *Decimal Fraction* to the *known parts* of the *Integer*;

If the *Decimal* given be part of a *Pound Sterling*, multiply it by 20 (the *Shillings* in a *Pound*) and the *Fractions* in the *Product* by 12 (the *Pence* in a *Shilling*) and the *Fractions* in this *Product* by 4, (the *Farthings* in a *Penny*.) Then the *whole Numbers* in the respective *Products* shew the *Shillings*, *Pence* and *Farthings* contained in the *Decimal* given, and the *Fractions* in the last *Product* are *Decimals* of a *Farthing*. *Example*; Suppose .25 a *Decimal Fraction* of a *Pound Sterling*, were given to be reduc'd as *aforesaid*; Multiply it by 20, the *Numbers* of *Shillings* in a *Pound*, the *Product* will be 5, *viz.* 5 s.

.25

20

Shillings ——— 5.00

So the *Value* of .60625 will be found to be 12 s. 2 $\frac{1}{2}$ d. As appears by the following *Example* :

.60625

Decimal Arithmetick.

15

	.60625
	20
Shillings	12.12500
	12
	35000
	12500
Pence	1.50000
	4
Farthings	2.00000

If the Decimal given be part of a *Beer-Barrel*, multiply it by 4, (the *Firkins* in a Barrel) and the Fractions in the Product by 9 (the *Gallons* in a Firkin,) and the Fractions in that Product by 8, (the *Pints* in a Gallon.) The *Whole Numbers* in the respective Products are equal to the *Firkins*, *Gallons* and *Pints* contained in the Decimal propounded.

Example.

Let .72 be a Decimal of a *Beer-Barrel*, to be reduc'd as aforefaid, it will be found to be 2 *Firkins*, 7 *Gallons*, 7 *Pints*, and .36 of a *Pint*, as appears by the Work.

	.72
	4
Firkins	2.88
	9
Gallons	7.92
	8
Pints	7.36

But

But these and many other *Arithmetical Questions* may be more readily resolv'd by the *Instrument*; a Description of which see in the *First Section* of the following Discourse.

The first thing to be done is to find out the value of the given number in the decimal system. This is done by multiplying the given number by the power of 10 which will make it an integer. For example, if the given number is 0.1234, multiplying it by 10000 will give 1234, which is an integer. This integer is then divided by the same power of 10 to get the decimal value. In this case, 1234 divided by 10000 gives 0.1234.

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STEREOMETRY,

OR, THE

ART of GAUGING

MADE EASIE

By the HELP of a New

SLIDING RULE.

SECT. I.

A Description of the RULE, and of the several Lines upon it, with their Use in some Questions in Arithmetick.

THE Rule consists of Three Parts, viz. a Rule of 12 Inches long, and two small Scales to slide in it, which may be drawn out, one towards the right Hand, and the other towards the left, till the whole be three Foot long.

The principal Lines on the Instrument are those commonly known by the Name of *Gunter's-Line*,
or

18 *Explanation of the Lines* Sect. I.

or *Line of Numbers*, which are here distinguished one from another by certain Letters, set at the end of the *Lines* towards the right hand.

Thus the *Lines D*, are each of them one *single Line of Numbers*, beginning at the End of the Rule towards the left hand, from thence continued to the other End.

The *Lines A B*, and *C*, are called *double Numbers*, each being *two Lines* or Radius's of Numbers, the *first* beginning at the left hand, and ending in the middle of the Rule where the *second Radius* begins, and is from thence continued to the end at the right hand.

The *Line E* is call'd *Triple Numbers*, being *three Radius's* of the Numbers; the *first* beginning at the left hand, and the *third* ending at the right hand.

This *Triple Line* is equal in length to the *double Lines*, and all to the *single Line*, for all the *five* begin and end at the same Point.

The *Line of Numbers* is (now) so well known to most Persons, that it may be thought a sufficient Description to have only said that *these* are *such*.

But for as much as this Instrument may be useful to some who (I presume) do not yet know what the *Line of Numbers* is, I shall therefore endeavour to explain it as followeth.

The *Line of Numbers*, is a *Line of Geometrical Proportions*, divided first into *Nine unequal Parts*, call'd *Primes*, which are distinguish'd by Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; and then each of these *Primes* are subdivided into *ten* other parts (according to the same Reason) call'd *Tenths*, and again each of those *Tenths* are subdivided, or at least suppos'd to be subdivided into *ten* other parts, according as the length of the *Line* will admit; as here the *Line D*, being about 11 Inches long, each *tenth* in the first *Prime* is

is really subdivided into *ten* parts, call'd *Centefms*, but betwixt the Figures 2 and 4, each *tenth* is divided out into five parts, and therefore each of those parts do signifie *two Centefms*: Again, from the Figure 4 to the end of the Line, every *tenth* is divided into two parts, each representing five *Centefms*: Lastly, each of these *Centefms* is also supposed to be divided into *ten* parts, which by some are called *Millions*; but a Line of this Length will not admit of this last division.

The Figures (1, 2, 3, 4, 5, 6, &c.) by which the *Primes* are distinguish'd, are all arbitrary Points, and may any of them represent so many entire *Units*, *Tens*, *Hundreds* or *Thousands*; or they may also represent so many *Tenth*, *Hundredth*, *Thousandth*, or *Ten Thousandth* parts of an *Unit*.

1. For *whole*, or *intire Units*; Let the first *Prime* or the Fig. 1. at the beginning of the Line D, represent one *Unit*, then shall all the Figures towards the right hand, (*viz.* 2, 3, 4, 5, &c. to 10.) represent so many *Units*, and the tenths in each *Prime*, will be *tenth parts*, and the *Centefms* in each of those tenths will be *hundredth parts* of an *Unit*.

Or, let 1 at the beginning of the Line represent 10 *Units*, then will each *Prime* forwards represent 10 times so many *Units* as the Figures express; thus, the Figures 2, 3, 4, 5, &c. will be 20, 30, 40, 50, &c.

And when one *Prime* represents 10 *Units*, every *tenth* in that *Prime* will be 1 *Unit*, and each *Centefm* in those tenths will be 1 tenth part of *Unit*.

Again, Let the first *Prime* represent 100, then the Figures 2, 3, 4, 5, &c. will represent 200, 300, 400, 500, &c. and therefore 10 at the end will be 1000; and according to this Supposition, 1 tenth in each *Prime* will be 10 *Units*, and in those tenths each *Centefm* will be 1 *Unit*.

2. For

2. For *Decimal Fractions* ; Let 10 at the end of the Line (at D) represent 1, then each *Prime* towards the left hand will be .1, and in those *Primes* each tenth will be .01, and these tenths each *Centefm* will be .001, part of on Unit.

To make this more plain, draw out the sliding Piece B, till 1 at the *beginning* of the Line B, stand exactly against 10 at the *end* of the Line A, for then you have a Line of Numbers four times repeated ; upon which, let 1 at the beginning of the Line A, represent 1 Unit ; then shall 1 in the *middle* of the said Line be 10; and 10 at the *end* thereof (or which is all one, at the *beginning* of the Line B.) represent 100 ; and by consequence 1 in the *middle* of the Line B will be 1000, and for the *same* reason 10 at the end of the said Line (which is also the *end* of the *Rad*) will be 10000.

But keeping the Rule as it now stands let 10 at the end of the fourth Radius (*viz.* at B.) represent 1, then shall each Prime in the said fourth Radius represent 1, in the third Radius .01, in the second Radius .001, and in the first .0001 part of an Unit.

So will 2 in the first Radius be .0002, in the second .002, in the third .02, and in the fourth .2 parts.

The *Numbers* and *Divisions* on the Lines being thus explain'd, it will not be difficult to find the Point upon the Line, where any Number given is represented.

As for Example ; Suppose the Numbers were 1895, for the first Figure thereof (*viz.* 1.) I count 1 at the beginning of the Line D, for the second Figure I count 8 tenths next following (that is 8 of the greater Divisions betwixt 1 and 2.) then from this Point forward I count 9 *Centesms* for the third Figure, and for

For the last Figure 5, I count half the next *Centefm*; so I find the Point *a g*, will represent 1895, and by the same Rule the Number 1715 will be found at the Point *w g*; hence observe

1. That on a Line of this length, only the *four* first Figures of any Number propos'd can be discover'd; for if the Number given were 189582, it would be represented at the same Point where the former 1895 was found (*viz.*) at *a g*.

2. That all Numbers which after the first Figure have nothing but *Cyphers* (as 20, 200, 2000, &c.) are all represented at the same Point.

So the Numbers 20, 200, 2000, are all represented by the Figure 2, at the beginning of the second *Prime*.

3. All Numbers consisting of *three* Figures, and having a *Cypher* in the *middle*, are found within the first tenth of that *Prime*, at which the first Figure of the Number given is found.

Example, Let the Number given be 308, for the first Figure I count 3 on the Line (which I find at the beginning of the third *Prime*) now there being a *Cypher* in the *second* place, I must not count any of the tenths; but for the last Figure 8 I count 8 *Centefms*, and that is the Point which represents 308.

4. All Numbers consisting of *four* places, and having *two* *Cyphers* in the *middle*, must be sought betwixt the beginning of the *Prime* unto which they belong, and the first *Centefm* of the same *Prime*; so 4005 being given, the first Figure (*viz.*) 4, is found at the beginning of the fourth *Prime*: Now there being *Cyphers* in the second and third places, I must not count any of the tenths or *Centefms*; but for the last Figure 5, I estimate 5 *Millions*, which is about the middle of the first *Centefm*, and that is the Point where 4005 is represented.

Note,

Note, *Decimal Fractions* and *mixt Numbers* are discover'd after the *same* manner as *whole Numbers*; for if the first Number 1895, were 18, 95, it will be found at the *same Point* (*viz.*) at *a g*; and by what hath been said, it is easie to find what Number is represented at any Point upon the Line, as will appear in the following Questions of Proportion, of which I come now to treat.

In *Arithmetick* I shall take notice of two sorts of Proportion, *Arithmetical* and *Geometrical*.

1. *Arithmetical* Proportion is when *divers Numbers*, being compared together, retain amongst themselves equal Differences, as these, 2, 4, 6, 8, &c. And this is either *continued*, as in the Numbers before propounded (which are also called *Arithmetical Progression*, or a *Rank* of Numbers *Arithmetically* proportional;) or *discontinued*, as in these, 2, 4, 8, 10, or the like.

2. *Geometrical* Proportion, is when *divers Numbers* being compared together, differ amongst themselves according to the same Rate or Reason, as these 2, 4, 8, 16, &c. for here, as 2 is half 4, so 4 is half 8, and 8 half 16; this is likewise either *continued*, as in those before propounded (which are also called *Geometrical Progression*, or a *Rank* of Numbers *Geometrically* proportional;) or *discontinued*, as in these, 2, 4, 16, 32; for as 4 is double 2, so is 32 double 16, but so is not 16, being compar'd with 4

Pro-

PROBLEM I.

Having two Numbers given, to find a third Geometrically proportional unto them, and to three a fourth, and to four a fifth, &c.

FInd one of the Numbers given upon the Line B, and set it against the other given Number on the Line A, then find the same Number upon B (which was last counted upon A) and against it you have this third proportional upon A; and against this third upon B, is the fourth upon A; in like manner, against the fourth upon B, you have the fifth on A, &c.

Example. Let it be required to find a third proportional to these two Numbers 2 and 4, which may bear the same proportion to 4, that 4 bears to 2.

Draw out the sliding Rod till 2 upon B stand against 4 upon A, then against 4 upon B is 8, (the third proportional) upon A; and against this third (viz. 8.) upon B, is 16 upon A; which is the fourth proportional: Likewise against 16 upon B, is 32 the fifth upon A, and against 32 upon B, is 64 the sixth proportional: But now proceeding forward, I find that 64 upon B, will reach beyond the end of the Line A, I therefore seek 64 towards the left hand upon B, and against it I find 128 the seventh proportional; and so proceeding further, you may find the eighth to be 256, the ninth 512, &c. Contrariwise, if it were required to find a third proportional to the same Numbers 2 and 4, which may bear the same proportion to 2, that 2 bears to 4.

Set

Set 4 in the second Radius upon A, to 2 upon B, then against 2 upon A (towards the left hand) is 1, the *third* proportional, and against 1 upon A is .5 the *fourth* upon B; also against this *fourth* (*viz.*) upon A, is .25, the *fifth* proportional on B, &c.

Multiplication by the Lines.

Problem II.

One Number being given to be multiplied by another, to find the Product.

IN Multiplication either of whole Numbers, mixt or decimal Fractions; the Proportion is :

As 1 is to the Multiplier;

So is the Multiplicand to the Product.

And the *Product* of any *two* Numbers shall have so *many* places as there be in *both* the Numbers given, except when the *lesser* of them do not exceed so *many* of the *first* Figures of the *Product*; then it will have *one* less.

1. Example : Let it be required to multiply 6 by 4.

Say, $[1 : 4 :: 6 : 24]$ which Analogy or Proportion may be read thus; *As 1 is to 4 So is 6 to 24.*

Therefore, Set 1 upon the Line B, to 4 on the Line A, and then against 6 upon B, is 24 upon A; which is the *Product* sought.

Note, The *Unit* or *first* Term may be taken upon either of the Lines A or B; but the *first* and *third* terms must be counted upon *one* and the *same* Line, and

and the *Second* on the *other* Line, where the *fourth* will also be found.

The Letters *A* and *B* may serve to distinguish the Lines.

2. Example: Let the two Numbers given be 68 and 26, to find the Product. The Proportion is, $1 : 26 :: 68 : 1768$.]

Therefore, Set 1 upon B, to 26 upon A, then against 68 upon B, is 1768 on A, which is the Product sought; Or,

Set 1 upon B, to 68 upon A, then against 26 upon B, is 1768 (on A) the Product. Therefore it matters not which of the Numbers given be made the *Multiplicator*; and note also, that the Product hath as many places as are in both the Numbers given, because the least of them (*viz.* 26) doth exceed so many of the first Figures of the Product, according to the Rule before given.

3. Example: Let 68 be multiply'd by 14. The Proportion is, $1 : 14 :: 68 : 952$.

Therefore, Set 1 upon B, to 14 upon A, then against 68 upon B, is 952 the Product upon A, and here the Product consists of one place less than there be in the two Numbers given, because the lesser of them (*viz.* 14) doth not exceed so many of the first Figures of the Product.

Now that the Product last found is 952, and not 95.2, nor 9.52, will thus appear: Set 1 upon B, against 14 upon A.

And then against	2	}	upon A is	28	}	upon B.
	3			42		
	4			56		
	5			70		
	6			84		
	6.8			95.2		

By this 'tis evident, that the Lines are in effect a Table of Multiplication, for having set 1 to the Multiplier, against any Multiplicand, you have the Product; so if 2 be multiply'd by 14, the Product is 28; if 3, the Product will be 42; if 4, the Product will be 56; if 5, 70, &c. Hence I conclude, that if the Multiplicand had been but 6.8, the Product would have been 95.2, but the Multiplicand being 68, the Product must be 952; for by taking away the prick, the Fractions in each are made whole Numbers.

When of *two* Numbers given to be multiply'd, the *one* consists of whole Numbers or *mixt*, and the *other* of Fractions only, make the *whole* or *mixt* number the *Multiplier*, and having set 1 against it, seek the Fraction towards the left hand, and against it you have the *Product*.

4. Example: Let the two Numbers be 27.5, and .8.

Set 1 on A, to 27.5 on B, and then against .8 (which being less than 1, I seek towards the left hand) on A, is 22, the Product on B.

And notwithstanding a Number of more than four places cannot be exactly expressed on a Line of this length, yet the Product of any Multiplication may be discover'd to six or seven places at least.

5. Example : Suppose I were to multiply 2482 by 54,
The proportion is,

$$1 : 54 :: 2482 : 134028$$

Therefore set 1 upon A to 54 upon B, and then
against 2, upon A, is 108 upon B : Now suppose,

$$2 \text{ be } \left\{ \begin{array}{l} 20 \\ 200 \\ 2000 \end{array} \right\} \text{ will be } \left\{ \begin{array}{l} 1080 \\ 10800 \\ 108000 \end{array} \right\}$$

So the Product will have six places, and against
2842 the Multiplicand, you may discern (upon the
Line) the four first of them, (*viz.*) 1340 ; the two
last may be found by multiplying (in ones mind) the
two last Figures of the *Multiplicand*, by the two last
of the *Multiplicator* ; for so shall you discover the two
last Figures of the *Product*, which in this Example
will be 28, which plac'd behind the four first (*viz.*)
1340, makes the Product compleat 134028.

I might here add more Examples, but these al-
ready given may serve, they containing (I think)
sufficient Directions for all the Variety that can hap-
pen in Multiplication.

Division by the Lines.

Problem III.

One Number being given to be divided by another, to find the Quotient.

IN Division both of whole Numbers and mixt, the Proportion is,
*As the Divisor is to 1,
 So is the Dividend to the Quotient.*

Which Quotient shall ever consist of so many Figures as the Dividend hath more than the Divisor, except when the Divisor does *not* exceed so many of the first Figures of the Dividend; then it shall have one place more.

1. Example: *Let it be required to divide 24 by 4;*
 The Proportion is, $4 : 1 :: 24 : 6$.

Therefore, Set 4 upon B, to 1 upon A, and then against 24 upon B, is 6 upon A, which is the Quotient sought.

2. Example: *Let it be required to divide 1768 by 26:* The proportion is, $26 : 1 :: 1768 : 68$.

Therefore, Set 26 upon B, to 1 upon A, and then against 1768 upon B, is 68 (the Quotient) upon A.

3. Example; *Suppose 952 were to be divided by 14:*
 The Proportion is, $14 : 1 :: 952 : 68$.

There-

Therefore, Set 14 upon A, to 1 upon B, then against 952 upon A, you have 68 upon B, which is the Quotient requir'd.

Observe here, the Dividend hath but one Figure more than the Divisor, yet the Quotient consists of two Figures, because the Divisor doth *not* exceed so many of the first Figures of the Dividend; but in the First and Second Examples the Quotients have but so many Figures as the Dividend hath more than the Divisor, because the Divisor doth *not* exceed so many of the first Figures of the Dividend, according to the general Rule above given.

This shews (in all Cases) how many Figures must be in the Quotient, the Value of the first of which may be found by the Rule given in the Introduction, *Pag. 8.*

On it will thus appear, that the Quotient in the last Example is 68, and not 6,8, nor .68: Set 14 upon A, to 1 upon B,

	28	3	2
And then	42	3	upon
against	56	4	upon
	70	5	B,
	84	6	
	952	6.8	

A is B,

This is but the Converse to the Third Example of Multiplication; and as by that it is manifest, that if 2 were multiplied by 14, the Product would be 28; if 3, it would be 42; if 4, 56, &c. So here it is evident, that if 28 be divided by 14, the Quotient will be 2; if 42, it will be 3; if 56, it is 4, &c. thus by reading on the Proportion from the

C 3

Di-

Divisor 14 : 28, 56, 70, &c. I find at last, that if the Dividend were but 95.2, the Quotient would be 6.8, but the Dividend being 952, the Quotient must be 68 ; for by taking away the prick, the Fractions in each are made whole Numbers.

By these Examples it is also evident that at once Setting of the Rule we both *Multiply* and *Divide*.

For if 14 be a Multiplier, set 1 on B, against 14 on A ; then against any Multiplicand upon B, you have the Product upon A ; as appear'd by the third Example of Multiplication.

And without moving the Rule, if you suppose 14 to be a Divisor, then against any Dividend upon A, you have the Quotient upon B ; as in the last Example of Division.

How by any Divisor to find a Multiplier, was shewn in pag. 13. of the *Introduction* ; it may also be perform'd by the *Lines* more readily.

Thus having a Divisor, to find a Multiplier :

Set the Divisor given upon A, to 1 upon B, and then against 1 (towards the left hand) upon A, is the Multiplier upon B. Example ; Suppose 25 were a Divisor given to find a Multiplier.

Set 25 upon A, to 1 upon B, and then against 1 (towards the left hand) upon A, is .04 the Multiplier sought.

By a Multiplier to find a Divisor : This is but the Converse of the former ; for having .04 a Multiplier upon B, set it against 1 upon A, and then against 1 upon B, is 25 the Divisor, as before.

Problem IV.

To reduce a Vulgar Fraction into a Decimal.

SET the Denominator (of the Vulgar Fraction given) upon A, to the Numerator thereof upon B, and then against 1 towards the left hand upon B, is the Decimal Fraction sought.

So $\frac{1}{4}$ will be found equal to this Decimal (*viz.*) .25 : For,

As 4 upon A, is to 1 upon B ; So is 1 upon A, to .25 upon B.

Also $\frac{3}{4}$ will be .75 ; for As 28 upon A, is to .21 upon B ; So is 1 upon A, to .75 upon B.

In like manner this Vulgar Fraction (*viz.*) $\frac{582}{960}$ is equal to this Decimal, *viz.* .60625 ; for set 960 upon A, to 582 upon B, then against 1 upon A, is .60625 upon B ; and so for any other.

Problem V.

A Decimal Fraction being given, to reduce the same into the known Parts of the Integer.

1. **I**F the Decimal be part of a Pound Sterling Set 1 upon B, to the Number of Shillings, Pence or Farthings contained in a Pound, upon A ; then seek the Decimal given upon B, (towards the left hand) and against it you have the Shillings, Pence or Farthings respectively contain'd in the Decimal given.

Example: Suppose .7625 were to be reduced into Shillings.

Set 1 upon B, to 20 (the Number of Shillings in a Pound) upon A, then against .7625 upon B, (towards the left hand) is 15.25, that is 15 s. and .25 parts of a Shilling, which is equal to 3 d.

2. To reduce the Decimal given into Pence: Set 1 upon B, to 240 (the Pence in a Pound) upon A, then against .7625 toward the left hand upon B, is 183, the Pence contained in .7625

3. But if the Decimal aforesaid were to be reduced into Farthings,

Set 1 upon A, to 960 (the Farthings in a Pound) upon B, and then against .7625 upon A, is 732; and so many Farthings are contained in .7625

Again, suppose .7625 were a Decimal of an Ale-Barrel, to be reduced into Gallons and Pints.

Set 1 upon A, to 32 (the Gallons of an Ale-Barrel) upon B, then against .7625 upon A, is 24.4, that is, 24 Gallons, and .4 tenth parts of a Gallon; to reduce this .4 into Pints; set 1 upon A, to 8 upon B, then against .4 upon A, is 3.2 Pints: So .7625 of an Ale-Barrel is equal to 24 Gallons, 2 Pints and 2 tenth parts of a Pint.

Problem VI.

Three Numbers being given, to find a fourth in a direct Proportion.

THIS is called the Rule of Three Direct, and by the Instrument is wrought thus;

Set the first Number given upon B, to the second upon A, and then against the third Number given upon B, is the fourth Number sought.

Ex-

Example: If 8 Quarters of Malt will make 20 Barrells of Strong-Beer, how many Barrells of such Beer will 22 Quarters make?

Set 8 upon B, to 20 upon A, and then against 22 upon B, is 55 upon A; and so many Barrells will 22 Quarters make, and according to this proportion 24 will make 60 Barrells, 28 will make 70, 32 Quarters will make 80 Barrells, &c.

Problem VII.

To three Numbers given, to find a fourth in an inversed Proportion.

THis is called the Rule of Three Inverse, in which observe, that if the third Number be greater than the first, then will the fourth be less than the second. And contrariwise, if the third Number be less than the first, the fourth will be greater than the second; and in either Cases the Rule is this:

Set the third Number upon A, the first (being of like Denomination) upon B, and then against the second Number upon A, you have the fourth upon B.

Example: If 8 Men do any piece of Work in 9 Days; In how many Days can 12 Men do the same Work?

Set 12 upon A, to 8 upon B, then against 9 upon A, is 6 upon B, which is the Answer. For 12 Men may do the same Work in 6 Days, that 8 Men will do in 9 Days.

But if the Question had been, *In how many Days can six Men do the same Work?* The Answer will be 12: For,

As 6 upon B, is to 8 upon A, So is 9 upon B, to 12 upon A.

Problem VIII.

Betwixt Two Numbers given, to find a Mean Geometrically Proportional.

SET one of the Numbers given upon C, to the same Number upon D, and then against the other given Number upon C, is the Geometrical Mean sought.

Example. Let the Numbers given be 50 and 72, to find a Geometrical Mean, &c.

Set 50 upon C, to 50 upon D, and then against 72 upon C, is 60 upon D. So 60 is a Geometrical Mean betwixt 50 and 72. Or thus:

Set 72 upon C, to 72 upon D, and then against 50 upon C, is 60 upon D, the Mean as before.

By the Pen thus:

Multiply one Number by the other, then extract the Square-Root of the Product, this Square-Root is the Geometrical Mean between the two Numbers given; thus, 72 multiplied by 50, is 3600, whose Square-Root is 60, the Geometrical Mean between 50 and 72.

Prob-

Problem IX.

To find the Square-Root of any Number under
1000000.

THE Extraction of Roots is one of the hardest Lessons in Arithmetick; yet by the help of this Instrument it may be perform'd with less trouble than any of the foregoing Problems: For if the Lines C and D be applied one to another, so as 10 at the end of D be even with 10 at the end of C; I say, the Lines thus applied are like a Table shewing the Square-Root of any Number by Inspection only; for against any Number upon C, you have the Square-Root thereof upon D, & contr.

Note, When the Number given consists of 1, 3, 5 or 7 places of Integers, seek it in the first Radius on the Line C, and against it you have the Root requir'd upon D.

Example. Let the Number given be 144; I find this on the first Radius on the Line C; and against it is 12, the Root sought. In like manner,

Against $\left\{ \begin{array}{l} 576 \\ 20736 \\ 1000000 \end{array} \right\}$ in the first Radius up-
on C, is $\left\{ \begin{array}{l} 24 \\ 144 \\ 1000 \end{array} \right\}$ the Root upon D.

2. When the Number given consists of 2, 4, 6, or 8 Places of Integers, find it in the second Radius upon the Line C, and against it you have the Root upon D.

Ex-

Example : Let 16 be the Number given.

I seek 16 in the second Radius upon C, and against it is 4, the Root upon D. In like manner;

Against $\left\{ \begin{array}{l} 30.25 \\ 2304 \\ 784996 \end{array} \right\}$ in the 2d. $\left\{ \begin{array}{l} 5.5 \\ 48 \\ 886 \end{array} \right\}$ the Root
Rad. up- upon
on C, is B.

Problem. X.

To find the Cube-Root of any Number under
1000.000.000.

PLace the Lines D and E, so as 10 at the end of D, be even with 10 at the end of E, then against any Number upon E, you have the Cube-Root thereof upon D. *Et Contr.*

Note, 1. When the Number given consists of 1, 4, or 7 places of Integers, find it in the first Radius of the Line E, and against it you have the Cube-Root sought. Example : Let the Number given be 3375 : I seek this in the first Radius in the Line E, and against it I find 15 upon D, which is the Cube-Root of 3375, and so is 212 the Cube-Root of 9528128.

2. When the Number given consists of 2, 5, or 8 places of Integers, find it in the second Radius upon E, and against it is the Root sought. Example : Suppose 35937 were propounded, find this in the second Radius upon E, and against it is 33, the Cube-Root upon D ; in like manner is 275 the Cube-Root of 20796875.

3. When

3. When the Number given consists of 3, 6, or 9 places of Integers, seek it in the third Radius, &c. and against it is the Cube-Root: Thus against 125 in the third Radius upon E, I find 5 the Cube-Root, and so likewise is 888 the Cube-Root of 700.227.072.

Lastly, to know how many places of Integers must be in the Cube-Root of any Number given.

Put a Point over the place of Units in the Number given, then omitting 2, point every third Figure toward the left hand, then tell how many Points, for so many places of Integers must the Cube-Root consist of.

SECT. II

Of a Superficies.

1. **A** Superficies is a Figure incompass'd about with a Line or Lines, and is either Round or Angular.

2. A Round Figure is that which is contained by one Round Line, and is either a Circle (as Fig. 1.) or an Ellipsis (as Fig. 2.)

3. An Angular Figure is that which doth consist of three or more Angles, from the Number of which they are denominated, as a Figure of three Angles is called a Triangle; of four, a Quadrangle, &c.

4. A Triangle is a Superficies comprehended by three Right Lines (as Fig. 3.) of these there are six Varieties, which I shall forbear to mention, they being all measured by one and the same Rule.

5. A *Quadrangle* is a Figure comprehended by four Right-Lines, and is either a *Parallelogram* or *Trapezium*.

6. A *Parallelogram* is a Figure whose opposite Sides are parallel, having equal distances from one another in all places, and is either Right or Oblique.

7. A *Right-Angled Parallelogram*, is that whose Angles are all Right, and is either a *Square* (as Fig. 5.) or an *Oblong* (as Fig. 4.)

8. The *Oblique-Angled Parallelogram*, is that whose Angles are all Oblique, and is either a *Rhombus* (as Fig. 7.) or a *Rhomboides* as Fig. 6.)

9. A *Trapezium* is a Quadrangular Figure, in which no two Sides are equal and parallel (as Fig. 8.)

10. Figures consisting of more Sides than four, are almost innumerable, but are reducible unto two sorts, *Regular* and *Irregular*, either of which are also called *Polygons*:

11. *Regular Polygons* are such, whose Sides and Angles are equal; they take their Names from the Numbers of their Sides, as that of five Sides is called a *Pentagon* (as Fig. 9.) that of six Sides, an *Hexagon*; &c. Of *Irregular Polygons* 'tis needless to say any thing, they being measured after the same manner as a *Trapezium*.

I should next proceed to shew the Use of the Rule in the measuring of Lines, Superficies, and Solids: But in order to this I must premise,

1. That every *Magnitude* must be measured by some known kind of *Magnitude* that is *Homogeneous* (or like) to it. A *Line* is measured by a *Line*, as one Lineal Inch, Foot or Yard, &c.

A *Superficies* is measured by a *Superficies*, as one Square Inch or Foot, &c. A *Solid* is measured by a *Solid*, as one Cubick Inch, one Cubick Foot, &c.

And

Sect. II. Of a Superficies.

39

And when it is known,

How many $\left\{ \begin{array}{l} \text{Lineal} \\ \text{Square} \\ \text{Cubical} \end{array} \right\} \left\{ \begin{array}{l} \text{Inches} \\ \text{or Feet} \end{array} \right\} \left\{ \begin{array}{l} \text{Line,} \\ \text{are con-} \\ \text{tained} \\ \text{in a} \end{array} \right\} \left\{ \begin{array}{l} \text{Superficies,} \\ \text{Solid,} \end{array} \right\}$

Then is the Quantity or Content of either of these kind of Magnitudes said to be known.

2. That the Measures of Capacity commonly used in England are of two Sorts, one for wet Commodities, as Beer, Cyder, Wine, &c. the other for dry Commodities, as Corn, &c.

And, I here take it for granted, that there are three several Gallons, whose Contents are allowed to be as followeth,

Ale $\left\{ \begin{array}{l} \text{Wine} \\ \text{Corn} \end{array} \right\} \text{Gallon} \left\{ \begin{array}{l} 232 \\ 231 \\ 168.8 \end{array} \right\} \left\{ \begin{array}{l} \text{Solid} \\ \text{Inches.} \end{array} \right\}$

Now from hence are derived the following Tables, shewing how many Solid Inches are contained in any of the other Measures respectively contained in them.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000
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A TABLE of Beer-Measure.

Inches.	Pints.	Quarts.	Gallons.	Firkins.	Kilderkins.	Barrels.
35 $\frac{1}{4}$						
70 $\frac{1}{2}$	2					
282	8	4				
2538	72	36	9			
5076	144	72	18	2		
10152	288	144	36	4	2	

A TABLE of Ale-Measure.

Inches.	Pints.	Quarts.	Gallons.	Firkins.	Kilderkins.	Barrel.
35 $\frac{1}{4}$						
70 $\frac{1}{2}$	2					
282	8	4				
2256	64	32	8			
4512	128	64	16	2		
9024	256	128	32	4	2	

By an Act of the First of William and Mary, 34 Gallons is the Barrel, both for Beer and Ale, in all places, except within the weekly Bills of Mortality.

A TABLE of Wine-Measure.

Inches.	Pint.	Quart.	Gallon.	Rundlet.	Hogshead.	Tericon.	Pipe.	Tun.
28 $\frac{7}{8}$	2							
57 $\frac{3}{4}$	4							
231	8	4						
4158	144	72	18					
14553	504	252	63	3 $\frac{1}{2}$				
19404	672	336	84	4 $\frac{3}{4}$	1 $\frac{1}{2}$			
29106	1008	504	126	7	2	1 $\frac{1}{2}$		
58212	2016	1008	252	14	4	3	2	

This Table shews, that in 1 Tun there are 2 Pipes, 3 Tericons, 4 Hogsheds, 14 Rundlets, 252 Gallons, 1008 Quarts, 2016 Pints, and 58212 solid Inches. The Tables for Beer and Ale are like this, and need no Explanation.

S E C T. III.

The Use of the Rule in measuring of Superficies ; and first, of a Circle.

THE Area or Superficial Content of a Circle is found by the Diameter or Circumference ; I shall therefore first shew how by either of these to find the other.

Problem. I.

The Diameter, or Circumference of a Circle, either being given, to find the other.

THE Circumference of that Circle whose Diameter is Unity (or 1) is * 3.1415926536 but for our purpose 3.141592, will suffice : Therefore, as 1 is to 3.141592, so is the Diameter of any Circle to the Circumference : By the Instrument thus :

* Foster's Proble-
mata Geomet. Varie,
Propos. 2.

Set 1 on the Line A, against 3.141592 on the Line B, then against any Diameter on the Line A, you have the Circumference on the Line B; and the contrary, thus : Against

These

These Dia- $\left\{ \begin{array}{l} 20 \\ 30 \\ 40 \\ 50 \end{array} \right\}$ you have these Circumferences, $\left\{ \begin{array}{l} 62.851 \\ 94.247 \\ 125.663 \\ 157.079 \end{array} \right\}$

Or, Contrariwise, against

These Circum- $\left\{ \begin{array}{l} 20 \\ 30 \\ 40 \\ 50 \end{array} \right\}$ you have these Diameters, $\left\{ \begin{array}{l} 6.366 \\ 9.549 \\ 12.732 \\ 15.915 \end{array} \right\}$

And so of any other.

Problem. II.

The Diameter of any Circle being given, to find the Area, (or any part thereof) in Inches, or in Ale or Wine-Gallons.

1. For the whole Area in Inches.

THE Area of a Circle is equal to the Fac^t, or Rect-Angle of half the Diameter into half the Circumference; that is to say, if half the Diameter be multiplied by half the Circumference, the Product will be the Area.

Thus, when the Diameter is 1, the Circumference is 3.141592, the half of this is 1.570796, which multiplied by half the Diameter (*viz.* .5) the Product will be the Area of that Circle, whose Diameter is 1, (*viz.*) .785398

The

The Area's of all Circles are in proportion one to another, as the Squares of their Diameters (2.12. of Euclid.)

Therefore, *As the Square of the Diameter of any Circle is to the Area that Circle;*

So is the Square of the Diameter of any other Circle to the Area thereof.

In the Circle above-mention'd the Diameter is 1, and the Area .785398: Now the Square of 1 being but 1, it must hold: *As 1 is to .785398, So is the Square of the Diameter of any Circle, to the Area thereof.* So .785398 is a fixed Multiplier; and if an Unit with Cyphers be divided by .785398, the Quotient will be 1.27324, a fix'd Divisor: and by either of these fix'd Numbers the Area of any Circle may be found, either by Multiplication or Division. For if the Square of any Diameter be

Multiplied { by } .785398 { Prod. } will be
Divided { } 1.27324 { Quot. }

the Area in Square Inches, Feet or Yards, according as the Diameter was measur'd by Inches, &c.

But with more expedition by the Instrument, Thus:

Set 1. (a Diameter) upon the Line D, to .785398 (the Area thereof) upon C.

The Rule being thus set, the Lines are like a Table of Circles Areas to all Diameters; for against any Diameter upon the Line D, you have the Area thereof upon C.

Sec. III.

Of a Circle.

43

Example: Let the Diameter be 20.

Set 1 upon D, to .785398 upon C, and then against 20 upon D, is 314.159, the Area required upon C; and as the Rule now stands, I also find, that if

The Diameter is $\left\{ \begin{matrix} 25 \\ 30 \\ 40 \end{matrix} \right\}$ the Area is $\left\{ \begin{matrix} 490.87 \\ 706.85 \\ 1256.63 \end{matrix} \right\}$ Inches.

Contrariwise, when the

Area is $\left\{ \begin{matrix} 300 \\ 400 \\ 500 \end{matrix} \right\}$ the Diameter is $\left\{ \begin{matrix} 19.54 \\ 22.56 \\ 25.23 \end{matrix} \right\}$

2. For the Area in Gallons.

The Area in Inches divided by 282 or 231, gives the Area in Ale or Wine-Gallons respectively, and so for any other Measure expressed in the former Tables; but without knowing the Area in Inches, the Area in Gallons may be found thus: Divide .785398

By $\left| \begin{matrix} 282 \\ 231 \end{matrix} \right|$ the Quotient will be $\left| \begin{matrix} .0027851 \\ .0033999 \end{matrix} \right|$ A. G. W. G.

The Quotients are the Area of a Circle, whose Diameter is 1, in Ale or Wine-Gallons, and are fixt Multipliers for finding the Area of all Circles in either of these Measures; for if the Square of the Diameter of any Circle be multiplied by either of these Numbers, the Product is the Area in Ale or Wine-Gallons respectively.

If

If you would effect this by Division, the several Divisors are thus found: Multiply the Divisor for finding the Area in Inches, viz. 1.27324

By $\left\{ \begin{matrix} 282 \\ 231 \end{matrix} \right\}$ the Product $\left\{ \begin{matrix} 359.05 \\ 294.11 \end{matrix} \right\}$ A. G.
will be $\left\{ \begin{matrix} 359.05 \\ 294.11 \end{matrix} \right\}$ W. G.

The Products are the Divisors sought. And the Square of the Diameter of any Circle divided by one of these, quotes the respective Area.

Having thus found the several fixed *Multiplicators* and *Divisors* for finding the Area of a Circle in Inches and Gallons, I shall here represent them all together in the following Table.

	Multiplicators.	Divisors.
Inches.	.785398	1.27324
Ale-Gallons.	.0027851	359.054
Wine-Gallons.	.0033999	294.118

But the Area of any Circle may be more readily found by the help of certain fixt Numbers, called *Gage-Points*, and these fixt Numbers are the Diameters of those Circles, whose Content at one Inch deep is equal to the respective Gallon to which they belong; thus the *Gage-Point* for the Ale-Gallon is 18.95, which is the Diameter of that Circle, whose Contents is 282, the Square Inches in the Ale-Gallons.

The several *Gage-Points*, are the Square-Roots of the Divisions last mention'd, and by the Rule are all found at once by the Lines C and D, for (setting these Lines even at the end)

Against $\left\{ \begin{array}{l} 159.05 \\ 294.11 \end{array} \right\}$ are these Gage-Points $\left\{ \begin{array}{l} 18.95 \\ 17.15 \end{array} \right\}$ A. G.
 these Divis. upon C, upon D, W. G.

To which are set the Letters *a. g.* and *w. g.* upon the Rule.

Now by these Gage-Points the Area of any Circle may be thus found :

For Ale-Gallons,

Set 18.95 (the Gage-Point for Ale-Gallons) upon D, to 1 upon C, then against any Diameter upon D, you have the Area upon C.

So if the Diameter were 40 Inches, the Area will be 4.45 Ale-Gallons: And the Rule being thus set, the Lines are in effect a Table of Circles Areas: For I likewise find, that if the Diameter

be $\left\{ \begin{array}{l} 45 \\ 50 \\ 52.5 \\ 58.6 \end{array} \right\}$ the Area will be $\left\{ \begin{array}{l} 5.64 \\ 6.96 \\ 7.67 \\ 9.56 \end{array} \right\}$ A. G.

The like for Wine-Gallons, by the proper Gage-Point.

Note, When the Area of any Circle is sought in Ale-Gallons, if the Diameter be more than 18.95, and less than 100, set the Gage-Point upon D, to 1, at the beginning of C; then against any Diameter, from the Gage-Point to 100, upon D, you have the Area upon C. Thus the Rule being set for Ale-Gallons, you will find that when the

Di.

Diamet. is	30	the Area will be	2.507	Gallons.
	40		4.456	
	60		10.026	
	80		17.825	
	100		27.851	

When the Diameter is less than the *Gage-Point*, or more than 100, then set the *Gage-Point* to 1, in the middle upon C; then against these

Diameters up- on D,	15	you have these Area's upon C,	.627	A. G.
	13		.471	
	10		.278	

And without moving the Rule, if 10 at the beginning of the Line D, be 100, the Area against it will be 27.851: So against this Diameter, viz. 200, the Area is 111.4 Gallons; against 300, it is 250.7, and so on to 600 Inches Diameter, against which you have 1002.6, &c. by this you may further observe, that if the Diameter be increased by Tens, the Area will increase by Hundreds: Thus

the Diameter	10	gives	.27851	Gallons.
	100		27.851	
	60		10.026	
	600		1002.6	

The like for Wine-Gallons by the proper *Gage-Point*.

Lastly, To find any part of the Area of a Circle, in Ale or Wine-Gallons.

Set the *Gage-Point* $\frac{1}{2}$, $\frac{1}{4}$, or any other part of 1; then against the Diameter you have the like part of the Area.

Ex-

Example: *Let it be required to find the third part of the Area of a Circle in Ale-Gallons.*

Set the *Gage-Point* to $\frac{1}{3}$ of 1, viz. .333, then against any Diameter you have $\frac{1}{3}$ of the Area; thus against 100 is 9.283, which is $\frac{1}{3}$ of 27.85, the whole Area.

Understand the like for Wine-Gallons, Ale or Beer-Barrels.

Problem III. Fig. 2.

The two Diameters (c d and e f) of an Ellipsis, being given, to find the Area or Content in Ale-Gallons.

AS the Square of the Diameter of a Circle is to the Area of that Circle; so is the Rect-Angle or Product of the greater and lesser Diameters of an Ellipsis, to the Area thereof. Therefore multiply the greater Diameter by the lesser Diameter, then that Product multiplied or divided by the fixed Multipliers or Divisors given (in pages 46, and 47,) gives the Area in Inches, Gallons or Barrels, according to the Number made use of.

Or thus, (by Problem VIII. Sect. I.) find a Geometrical Mean Proportion between the greater and lesser Diameters, for this Mean is the Diameter of a Circle, whose Area is equal to the Area of the Ellipsis.

Example. Fig. 2.

Let the greater Diameter $c d$ be 72 Inches, and the lesser $e f$, 50; by the foregoing Rule, the Geometrical Mean between them will be found to be 60, the Diameter of a Circle equal to the *Ellipsis*, and the Area of a Circle, whose Diameter is 60, will be found to be 10.02 Ale-Gallons. But the Area of an *Ellipsis* may be more easily found by the Rule, thus;

Set 359.05 upon B, to one of the Diameters (suppose 50) upon A; then against the other Diameter 72 upon B, you have the Area upon A, which in this Example will be 10.02, Ale-Gallons, the Content of this *Ellipsis* at one Inch deep: The like may be done for Wine-Gallons, if instead of 359.05, you take 294.11.

Problem IV.

To find the Area or Content of a Triangular Superficies.

IN all Right-lined Triangles, Multiply half the longest Side by the Perpendicular (which is always the nearest Distance from the Base to the opposite Angle;) or Multiply half the Perpendicular by the whole Base; the Product will be the Area required.

Example. Let Fig. 3. represent a Triangular Back or Cooler, whose longest Side $a b$, is 260 Inches, and the Perpendicular Line $c o$, 110 Inches, the half of $a b$ is 130; this multiply'd by 110, gives 14300 for the Area in Square Inches, and this divided

Sect. III. Of a Parallelogram. 51

vided by 282, quotes 50.7, the Area in Ale-Gallons ; or by the Rule at one Operation, thus :

Set 282 upon B, to 130 upon A ; then against 110 upon B, is 50.7 upon A, the Content as before.

To measure Quadrangular Figures:

Problem V.

In any Right-angled Parallelogram, be it Square or Oblong ; The Product of any two Sides, including one and the same Angle, is equal to the Area or Content.

1. Example.

IN Fig. 5. the Sides hi and ik , are equal ; suppose each be 138 Inches ; this multiply'd by itself, is 19044, the Area in Square Inches : For the Area in Gallons by the Rule,

Set 282 upon B, to 138 upon A ; then against 138 upon B, is 67.53, the Area in Ale-Gallons at one Inch deep.

2. Example, Of an Oblong (Fig. 4.)

Suppose ns , 130 Inches, and su , 180, the Product of these (being multiply'd) is 23400, the Area in Inches: But for Ale-Gallons, set 282 upon B, to 180 upon A ; then against 130 upon B, is 82.97 Ale-Gallons, the Area required.

D 2

Prob.

Problem VI.

In any Oblique-Angled Parallelogram (be it a Rhombus or a Rhomboides) Multiply the shortest Distance between the two longest Sides, by one of the said Sides, the Product is the Area in Inches.

1. Example. Of a Rhombus (Fig. 7.)

Suppose the Side pt , were 130 Inches, and the distance qy , 108; these multiplied, the Product will be 14040, the Area in Inches: But by the Rule thus:

As 182 is to 108, So is 130 to 49.78, the Area in Ale-Gallons at one Inch deep.

2. Example. Of a Rhomboides (Fig. 6.)

Let the Side xz , be 260, and the Distance tw , 108, the Product of these is 280, the Area in Inches. By the Rule:

Set 282 upon A, to 260 upon B; then against 108 upon A, is 99.57, the Area in Ale-Gallons.

Problem. VII. Figure 8.

To find the Area or Content of a Trapezium.

A RightLine drawn from one of the Acute Angles to the Angles opposite (as the Line ab) will divide the Trapezium into two Triangles, the Area's of which are equal to the whole Trapezium, and may be found by the 4th Problem of this Section: Or thus; Multiply the Sum of the two Perpendiculars (fe and cr) by the half of ab , the Product is the Area of the Trapezium: This is so plain, that it needs no Example.

All other irregular Right-lined Figures consisting of more than four Sides, must be divided into Triangles (which will be ever less by two than the Number of Sides) and then the Area's of all those Triangles are equal to the Area of the whole Figure.

Problem VIII.

The sides of any Regular Polygon being given to find the Area.

IN any Regular Polygon, multiply half the Sum of the Sides by the Perpendicular (or nearest Distance from the Center to one of the Sides) the Product is the Area.

Example: In a Pentagon (or Figure of five equal Sides) as Fig. 9.

Suppose each Side be one Inch, to find the Perpendicular (CA) we have given $AB=5$, and the Angle at $C=36$ Degrees.

D 3

Therea

Therefore,

As the Sine of $C = 36^{\circ}$ d.	Log.
Is to the Side $AB = 5$	Co. Ar. 0.230781
So is the Sine of $B = 54^{\circ}$ d.	0.698970
	<u>9.907957</u>
To the Side $CA = .68819$	.837708

which is the Perpendicular sought; and this .68819 multiplied by half the Sum of the Sides, viz. 2.5, the Product is 1.72047, the Area of a Pentagon, whose Side is one Inch; and by this method I find the Area of all the other Polygons which are express'd in the following Table. In the first Column of which you have the Names of the Polygons; in the second their Area's in Inches; each of which being divided by 282, quotes the Area in Ale-Gallons, which are the Numbers in the third Column; and in the 4th and 5th Columns you have the respective Divisors.

Names

Names of the Polygons.	Multipliers, Being the Area's in Inches and Gallons when the Side is one Inch.		Divisors, Being the Square of the Sides when the Area is one Inch or one Gallon.	
	Inches.	Ale Gall	Inches.	Ale Gall
Trigon	.433013	.0045355	2.309401	.6518251
Tetragon	1.000000	.0035461	1.000000	.281.000
Pentagon	1.720477	.0261010	.581234	.163.908
Hexagon	2.598076	.0092130	.384900	.108.342
Heptagon	3.633959	.0128864	.275182	.77.601
Octagon	4.828427	.0171221	.207107	.58.404
Nonagon	6.181824	.0219214	.161764	.45.618
Decagon	7.694209	.0272844	.129968	.36.651

Now having the Area (both in Inches and Ale-Gallons) of these Polygons when the Side is Unity, the Area of each may be readily found, the Side being given: For,

As the Square of the Side of any Polygon, is to the Area of that Polygon;

So is the Square of the Side of any other like Polygon, to the Area thereof: Therefore,

If the Area of any Polygon (expressed in the Table) be desired, in Inches or Ale Gallons.

Multiply or divide the Square of the Side by the respective Multiplier or Divisor, the Product or Quotient will be the Area sought.

Example: For the Area in Inches, suppose the Side of a Pentagon be 50 Inches; the Square is

D 4.

2500.

2500. This multiplied by 1.720477, or divided by .581234, gives 4301.2, the Area in Inches.

For the Area in Ale Gallons, If the Square of 50 (*viz.* 2500) be multiplied by .006101, or divided by 163.908, the Product or Quotient will be 15.252, the Area sought.

To find the Side by the Area given.

Multiply or divide the given Area by the proper Multiplier or Divisor; the Product or Quotient will be the Square of the Side, whose Square-Root is the Side sought.

Thus the Area of a *Pentagon* is 15.252 Ale-Gallons, This multiplied by 163.908, or divided by .006101, gives 2500, whose Square-Root is 50, the side sought.

But the Lines C and D, will effect this with much more expedition; for at once setting of the Rule you have the Area in (Inches or Ale-Gallons) to any given Side, or the Side to any Area given.

Example: For the Area in Ale-Gallons, set 1 upon D, to (the Area in Gallons when the Side is 1, *viz.*) .006101 upon C; then against any Side upon D, you have the Area upon C, and the contrary.

Thus against 50 upon D, is 15.252 the Area in Ale-Gallons; Also against 22 Gallons, an Area upon C, you have 60 the Side upon D, &c.

S E C T. IV.

The Use of the Rule in the Mensuration of Solids; and first, of a Prism.

A Prism is a Solid contained by several Planes, two of which being opposite, are called the Bases, and these are equal, Parallel alike, and alike situate; but the other Planes are Parallelograms, in which a Right Line may be every where applied from Base to Base: Under this Name Prism is comprehended that Solid of two Circular Bases, usually called a Cylinder.

Problem I.

To find the Solid Content of a Prism.

Multiply the Area of the Base by the Perpendicular Height, the Product is the Content sought.

Example: Let Fig. 11. represent a Brewer's Tun, in the form of a Square Prism, whose Base g, h, k, i , is equal to Fig. 5. each Side being 138 Inches, and the Perpendicular ($s i$) 30 Inches, how many Ale-Gallons or Beer-Barrels will this Tun contain?

Set 282 upon B, to 138 upon A; then against 138 upon B, is 67,534 upon A, the Content in Ale-Gallons at one Inch deep; this multiplied by 30, the whole Depth, gives 2026.02, the whole Content in Ale-Gallons; this divided by 36, quotes 56.278, the Content in Beer-Barrels.

The Fraction .278 (being reduced) is 10 Gallons, and .188 parts. For,

As 1.000 : 36 :: .278 : 10.188.

2. Example. Of a Round Prism or Cylinder.

Let Fig. 12. represent a round Tun, whose Diameter (*n o*) at Top is equal to (*s y*) the Diameter at Bottom, each being 120 Inches, and the Altitude (*e n*) 36 Inches: How many Gallons or Beer-Barrels may this Tun contain. First, for Gallons,

Set the Gage-Point (*a g*) to 36, the Tun's Depth upon C, then against 120 the Diameter, upon D, is 1443.6, the Content in Ale-Gallons.

2. For the Content in Beer-Barrels: 1443.6 divided by 36, quotes 40.1, the Content in Barrels.

Suppose I come to this Tun, and find the Liquor 9 Inches deep: How many Gallons are then contained in the Tun?

Set the Gage-Point to 9; then against 120 is 360.95 Gallons.

Problem II.

The Diameter, Depth and Content of any Cylindrical Tun, any two being given to find the Third.

IN this Problem are three Questions, all resolved at once setting the Rule.

1. By the Depth and Content, to find the Diameter.

Example : Suppose the Depth 40 Inches, and the Content 1800 Ale-Gallons. What is the Diameter ?

Set 40 the Depth upon C, to the Gage-Point upon D: then against 1800 the Content upon C, is 127.1 Inches, the Diameter sought.

2. By the Diameter and Content, to find the Depth, without moving the Rule : Say,

*As 127.1 the Diameter,
is to 1800 the Content ;
So is the Gage-Point ,
to 40 the Depth.*

} upon { $\begin{matrix} D \\ C \\ D \\ C \end{matrix}$

3. By the Depth and Diameter to find the Content, the Rule standing as before : Say,

*As the Gage-Point
is to 40 the Depth ;
So is 127.1 the Diameter,
to 1800 the Content*

} upon { $\begin{matrix} D \\ C \\ D \\ C \end{matrix}$

Note,

Note, As a Circle is the Base of a Cylinder or round Prism; so a Triangle, Quadrangle, or any other plain Superficies, may represent the Base of a Prism; for if there be Planes erected perpendicularly upon the Lines which incompass any such Superficies, they will generate a Solid which may be called a Prism; and the Content of any such Solid is gotten by multiplying the Area of the Base by the Altitude, or distance from one Base to another.

S E C T. V.

The Use of the Rule in the Mensuration of a Sphere, and its Frustums.

Problem. I.

The Diameter of a Sphere or Globe being given in Inches, to find the Content thereof in Inches or Gallons, both Ale and Wine.

A Sphere is two third Parts of a Cylinder, whose Diameter and Altitude are equal to the Diameter of the Sphere.

But if the Diameter and Altitude of a Cylinder, be 1, the Area of the Base, which is also the solid Content (for 1 doth not multiply) is .785398: Therefore $\frac{2}{3}$ of .785398, viz. .523598; is the Content of a Sphere, whose Diameter is Unity or 1.

More—

Moreover, all Bodies are in proportion one to another, as the Cube of their like Sides. Therefore,

As the Cube of the Diameter of any Sphere, is to the Content of that Sphere;

So is the Cube of the Diameter of any other Sphere, to the Content thereof.

Now if the Diameter be 1, the Cube thereof is but one: Therefore,

As 1 is to .523598;

So is the Cube of the Diameter of any Sphere to the Content.

Example. Let the Diameter of a Sphere be 20 Inches, and the Content required in Solid Inches.

By the Lines D and C, on the Rule, the Cube of 20 will appear to be 8000; this multiplied by .523598, is 4188.784 the Content sought.

For the Content in Gallons.

If .523598 be divided

By $\left\{ \begin{smallmatrix} 282 \\ 231 \end{smallmatrix} \right\}$ quotes $\left\{ \begin{smallmatrix} .0018567 \\ .002266 \end{smallmatrix} \right\}$ the Con- } A. G.
tent in } W. G.

Now if the Cube of the Diameter of any Sphere be multiplied by either of these Numbers, the Product will be the Content in Ale or Wine-Gallons respectively: Thus, the Cube of 20 the Diameter (*viz.*) 8000, multiplied by .001856, is 14.848 the Content in Ale-Gallons; and the like may be done for Wine.

But

But the Content of any Sphere may be more readily found by the Instrument. Thus,

Set 1 upon D, to .523598 upon E; then against any Diameter upon D, is the Content upon E: So against 20 (the Diameter above-mentioned) upon D, is 4188.784 the Content in Inches, as before: In like manner, if the Diameter were 30, the Content will be 14137.14, &c.

For Ale-Gallons thus:

Set 1 upon D, to .001856 upon E; then against any Diameter upon D, is the Content upon E; as against 20 upon D, is 14.84, the Content as before; and without moving the Rule, I find that if the

Dia- { 30 } the Con- { 50.13 }
ter be { 40 } tent is { 118.78 } A.G.

Problem II.

Having the Altitude of the Frustrum of a Globe, together with the Diameter of its Base, to find the Altitude of the other Frustrum; or (which is all one) the Remainder of the Globe's Axis.

The RULE is,

Divide the Square of the Semidiameter of the Frustrum's Base, by the Altitude of either of the Frustrums, the Quotient will be the Altitude of the other Frustrum.

Example. In Fig. 10. let cb , the Altitude of the Frustrum, be 6 Inches, and dce , the Diameter at the Base, 24, the half of this (*viz.* dc) is 12, which Squared, is 144, this divided by the Altitude cb , *viz.* 6, the Quotient is 24, equal to ca , the Altitude of the other Frustrum. Note also, that a Geometrical Mean Proportion between ac and cb , is equal to cd , which doubled, is de , the Diameter of the Frustrum's Base. Euclid 6, 13.

Pro-

Problem. III.

Having the Altitude of the lesser Frustrum of a Sphere, and the Diameter of its Base, to find the Content in Inches or Gallons.

Multiply the Square of half the Diameter of the Base by three times the Altitude; and to this Product add the Cube of the Altitude, the Sum multiply'd

By $\left\{ \begin{array}{l} .523598 \\ .0018567 \end{array} \right\}$ Gives the Con- | Inches.
tent in | Ale-Gall.

Example: In Fig. 10. let $d c e$, the Diameter of the lesser Frustrum ($c d b e$) be 24 Inches, and $c d$ its Altitude, 6 Inches; What is the Content in square Inches?

$c e$ squared =

144

3 times $b c$ =

18

1152

144

2592

Cube of $b c$ =

216

2808

2808 multiplied

Content.

By $\left\{ \begin{array}{l} .523598 \\ .0018567 \end{array} \right\}$ gives

$\left\{ \begin{array}{l} 1470.263, \text{ Inc.} \\ 5.2137, \text{ A. G.} \end{array} \right\}$

The

The Content of the lesser *Frustum* taken from the Content of the whole Sphere, leaves the Content of the greater *Frustum*.

S E C T. VI.

The Use of the Instrument in the Mensuration of Pyramids and their Frustums.

A Pyramid is a Body contained under several Planes, set upon one Right-lin'd Base, and meeting in a Point at the Top, which is called the Vertex, also in each of these Planes a Right Line may be every where applied from the Base to the Vertex.

And under this Name *Pyramid* is comprehended that *Pyramidical Body* (whose Base is a Circle) commonly called a *Cone*.

Phoblem I.

To find the Solid Content of a Pyramid:

A Pyramid is one third part of a *Prism*, which hath the same Base and Altitude. Therefore,

Multiply the Area of the Base by $\frac{1}{3}$ of the Altitude, or $\frac{1}{3}$ of the Area by the whole Altitude; in either Cases the Product will be the Content of the Pyramid.

Ex-

1. Example.

In a Square Pyramid (as Fig. 17.) suppose the side of the Base $x p$, be 112 Inches, and the Altitude $z s$, 90 Inches, what is the Content in Ale-Gallons?

Set 182--- upon B | to 112 | upon
then against 112 | is 44.482 | A,

the Area of the Base in Ale-Gallons. Then set 1, upon A, to 30 upon B, and against 44.482 upon A, is 1334.46, the Content in Ale-Gallons.

2. Example.

There is a round Pyramid or Cone (as Fig. 18.) suppose the Diameter at the Base $x n$, be 100 Inches, and the Altitude $x s$, 180; How many Ale-Gallons may this Cone contain?

Set the Gage-Point for an Ale-Gallon, to $\frac{1}{4}$ of the Altitude (*viz.* 60) upon C; then against 100 upon D, is 1671, the Content required.

Problem II.

To find the Solid Content of the Frustum of a Pyramid.

1. **B**y the Rules foregoing, find the Area of the greater, and also of the lesser Base.

2. Find a Geometrical Mean Proportion betwixt these two Area's.

3. The Sum of these three Numbers being multiplied by one third part of the Altitude, the Product is the Content of the Frustum sought.

Example. Let $a c c o$ (in Fig. 18.) represent the Frustum of a round Pyramid or Cone, $a a$ the Diameter of the greater Base is 108 Inches, $c c$, the Diameter of the lesser Base, 81, and $a u$, the Altitude, 36 Inches, to find the Content in Ale-Gallons.

1. Set the Gage-Point to 1 at the beginning of the Line C; then against 108 upon D, is 32.48, the Area of the greater Base in Ale-Gallons.

2. Set the Gage-Point to 1 at the beginning of the Line C; then against 81 upon D, is 18.27, the Area of the lesser Base.

3. Set 32.48 upon C, to 32.48 upon D; then against 18.27 upon C, is 24.36, the Geometrical Mean between the two Area's.

So the Area of the greater Base is 32.48

The Area of the lesser is 18.27

A Geometrical Mean betwixt the 2 Area's is 24.36

The Sum is 75.11
This

This multiplied by $\frac{2}{3}$ of the Altitude (*viz.* 12.) gives 901.32, the Content of the Frustrum sought.

Or by the Pen thus:

1. Square the Diameter of the greater Base.
2. Square the Diameter of the lesser Base.
3. Multiply one of these Diameters by the other. To this Product add the two Squares, and multiply the Sum by the Altitude: The last Product, divided by 1077, quotes the Content in Ale-Gallons.

S E C T. VII.

The Use of the Rule in Gauging of Brewers Tuns.

THese Vessels are in several Forms: The most usual may be considered under some of these Varieties.

I. *Such whose Bases are equal.*

And if the equal Bases be alike (that is, both Round, both Square, &c.) and Parallel, and the Sides of the Tun streight from one Base to the other. these Tuns are called *Prisms*. See Problem I. of this Section.

II. *Such*

II. Such whose Bases are unequal.

1. If the unequal Bases be alike (suppose both Rectangular Parallelograms, both Elliptical, &c.) Parallel, Proportional, and alike situate, and the Sides of the Tun streight, such Tuns are the Frustums of Pyramids. See Problem II. of this Section.

2. If the unequal Bases be unlike, suppose one Round, and the other Elliptical, or if they be alike but not proportional, or if they be both alike and proportional, yet if they be not alike situate, the Tun is called a *Prismoid*, in which 'tis still supposed the Bases are parallel, and Sides of the Tun streight. See Problem VII.

Note, When the Bases are proportional, it will be,

As the Length of the greater Base, is to the Length of the lesser :

So is the Breadth of the greater Base, to the Breadth of the lesser.

And the Bases are said to be alike situate, when the Length of the greater and lesser Bases are both in the same Plane.

Problem I. Fig. 11.

There is a Tun whose Bases are both Equal, Parallel, and Square, each Side of either Base being 138 Inches, and the Depth 33 Inches; what is the Contents of this Tun in Ale-Gallons? Answer, 2228.62. For,

SET 282 upon B, to 138 upon A; then against 138 upon B, you have 67.534 upon A, the Content at one Inch deep; this multiplied by the whole Depth, viz. 33, gives 2228.622 the Content required. Or thus:

Set 16.79 upon D, to 33 (the Tun's Depth) upon C; then against 138 upon D, is 2228.622 upon C, the Content as before.

Note, 16.79, is the Gage-Point for square Vessels, it being the Square-Root of 282, the Inches in an Ale-Gallon.

It is needless to give more Examples of this sort of Tuns; for let the Bases be in any Form whatsoever, if the Tun have the Qualifications above-mention'd, the Content is found by multiplying the Area of the Base by the Depth; and if the Bases are round, as in most Tuns they are, the Content may be found at one Operation, as hath been shewn in the I. and II. Problem of Sect. IV.

I. Sect. VII. Of Gauging of Tuns.

71

Of Tuns, whose Bases are unequal.

Problem II. Flg. 16.

There is a Tun (as a, b, c, d, e, f, g, h,) whose Bases are Rectangular Parallelograms, Unequal but Parallel, and alike situate, and the Sides Proportional (d a) the Length of the greater Base, is 100 Inches, and (d c) the Breadth 80 ; (h e) the Length of the lesser Base 85, and (h g) the Breadth 68, and the Depth of the Tun 30 Inches : How many Ale-Gallons may this Tun contain ?

FOR all Tuns of this sort, this is a general Rule.

Take the Area of the greater Base, the Area of the lesser Base, and a Geometrical Mean Proportional betwixt the two Area's ; the Sum of these three multiplied by one third part of the Tun's Depth, gives the Content.

Example either by the Pen, or the Lines A and B upon the Rule.

As 282 is to $d a$ 100 ; So is $d e$ 80 to
the Area of the greater Base, viz. 28.368

As 282 is to $h e$ 85 : So is $h g$ 68 to the
Area of the lesser Base, 20.496

A Geometrical Mean between these
two Area's will be found (by Prob.
VIII. Sect. 1.) to be 24.112

The Sum is 72.976

This multiplied by one third part of the Tun's
Depth, viz. 10, gives 729.76 the Content in Ale-
Gallons.

Problem III. Fig. 16.

There is a Tun whose Bases ($i l k m$ the greater, and
 $n q o p$ the lesser) are both Elliptical, Parallel,
Proportional, and alike situate ; the greater and
lesser Diameters of each Base being equal to the
Lengths and Breadths of the Bases of the former
Tun, and the Depth the same ; How many Ale-
Gallons will this Tun contain ?

THis Elliptical Tun may be conceived to be in-
scribed in the Tun, whose Content was found
in the last Problem : all the given Dimensions are
the same in both, and the Operation will be the
same too ; for the first Term in the Proportion we
take 359.05 instead of 282. See the Work.

As 359.05 is to 100; So is 1780 } 12.281
to the Area of the greater Base.

As 359.05 is to 85; So is 1780 } 16.098
the Area of the lesser Base

The Geometrical Mean between these two
Area's will be found (by Prob. VIII.
Sect. I.) to be } 18.938

The Sum is 57.317

This multiply'd by one third part of the Depth,
viz. 10, gives 573.17 the Content of this Elliptical
Tun, in Ale-Gallons.

Problem IV.

There is a Tun (as Fig. 17.) whose Bases are
both Square, but unequal (x p) the Side of
the greater Base, is 108 Inches, (r o) the
Side of the lesser Base 93, and the Depth, (sa)
30 Inches, what is the Content in Ale-Gal-
lons?

THIS Tun being the Frustum of a Pyramid
may be Gauged by the General Rule in Pro-
blem II. of this Section. Or thus:

E

1. Square

1. Square the Side of the greater Base, and also the Side of the lesser Base.

2. Multiply the greater Side by the lesser, and to the Product add the two former Squares.

3. The Sum of these three being multiplied by $\frac{1}{3}$ of the Depth, gives the Content in Square Inches, which divided by 282, the Quotient is the Content in Ale-Gallons.

Example: By the Instrument the Square } 11664
of x p 108 is —————

The Square of o r 93 is ————— 8649
108 multiplied by 93, is ————— 10044

Sum is ————— 30357
Multiplied by $\frac{1}{3}$ Altitude ————— 10

The Product, viz. ————— 303570

This divided by 282 quotes 1076.48. the Content sought.

There is a Tun (in Fig. 1) with a top of 108 inches, and a bottom of 93 inches, and a depth of 10 inches. The greater Base is 108 inches, and the lesser Base is 93 inches, and the Depth is 10 inches. What is the Content in Ale-Gallons?

This Tun being the Product of a Tun, may be Gauged by the General Rule in Prob. II. of this Section. Or thus.

1. 282

10

Prob-

Problem V. Figure 18.

There is a Conical Tun whose Bases are both Circular; (a o) the Diameter of the greater Base is 108 Inches, (c e) the Diameter of the lesser Base 93, and (u n) the depth 30 Inches: What is the Content of this Tun in Ale-Gallons?

THE Dimensions being the same as in Problem IV. of this Section, proceed as is there directed, the last Product will be 303570; this divided by 359.05, quotes 845.48 the Content sought,

Or thus by the Rule.

Subtract the Diameter of the lesser Base from the Diameter of the greater, and add half the Difference to the lesser, the Sum is the Diameter in the middle of the Tun's Depth; which found,

1. Set the Gage-Point (a g) to the Tun's depth upon C.

Then against the Diameter upon D, is a fourth Number upon C, which keep.

2. Set the Gage-Point to $\frac{1}{2}$ of the Tun's depth upon C.

Then against $\frac{1}{2}$ the Difference of the Diameters upon D, is a fourth Number, which added to the fourth Number first found, is the Content of the Tun sought.

Example. Figure 18.

a a the Diameter of the greater Base is 108

c c the Diameter of the lesser Base is 93

The Difference is 15

The half Difference is 7.5, this added to 93 makes 100.5 for the Diameter in the middle of the Tun's depth: *Therefore,*

1. Set the Gage-Point to 30 the Tun's Depth, then against 100.5 the Diameter is 843.91, which keep.

2. Set the Gage-Point to $\frac{1}{2}$ of the Tun's Depth, that is, 10; then against 7.5 the Semidifference of the Diameters, is 1.56: This added to 843.91 (the Number first found) the Sum is 845.47; the Content sought; which agrees very well with the former Rule.

Problem IV.

To find the Content of a Pyramidal Tun whose Bases are in the Form of any of the first 10 Regular Polygons, one Side of each Base, and the Tun's Depth being given.

THE Area of each Base of any such Tun may be found by Problem VIII. Section III.

A Geometrical Mean betwixt these Area's may be found by Problem VIII. Section I.

The Sum of the two Area's and the Geometrical Mean between them being multiplied by $\frac{1}{3}$ of the Tun's Depth, gives the Content. Or thus:

By

By the Rule in the last Problem, square the side of each Base, multiply the greater Side by the lesser; then multiply the Sum of these three by $\frac{1}{3}$ of the Tun's Depth. Lastly, multiply or divide this last Product by the proper Multiplier or Divisor [as per Table page 55.] the Product or Quotient will be the Content in Ale-Gallons.

Example. Let there be a Pyramidal Tun, whose Bases are both in form of a Pentagon (or Figure of five equal sides) and let the side of the greater Base be 108, the side of the lesser 93, and the depth 30 Inches; all the same as in the last Problem; Therefore according to the Rule and Example there laid down, the last Product will be 303570: This multiplied by .006101, or divided by 163.908, [vid. Table, page 55.] gives 1852.08.

To find the Content of any Tun, whose Parallel Bases are Rectangular Parallelograms, or Ellipses, the sides of the Tun being straight from the Top to the Bottom, whether the Bases be alike or unlike, proportional or disproportional, alike situated or inverted, by one general Rule.

Rule. Square the side of the greater Base, multiply the side of the lesser Base by the side of the greater Base, add the square of the side of the lesser Base to the product, multiply the sum by the Depth of the Tun, and divide the product by 163.908, the Quotient will be the Content in Ale-Gallons.

Example. Let there be a Tun, whose Parallel Bases are Rectangular Parallelograms, the side of the greater Base be 108, the side of the lesser Base be 93, and the Depth be 30 Inches; all the same as in the last Problem; Therefore according to the Rule and Example there laid down, the last Product will be 303570: This multiplied by .006101, or divided by 163.908, [vid. Table, page 55.] gives 1852.08.

Example. Let there be a Tun, whose Parallel Bases are Ellipses, the side of the greater Base be 108, the side of the lesser Base be 93, and the Depth be 30 Inches; all the same as in the last Problem; Therefore according to the Rule and Example there laid down, the last Product will be 303570: This multiplied by .006101, or divided by 163.908, [vid. Table, page 55.] gives 1852.08.

Example. Let there be a Tun, whose Parallel Bases are Ellipses, the side of the greater Base be 108, the side of the lesser Base be 93, and the Depth be 30 Inches; all the same as in the last Problem; Therefore according to the Rule and Example there laid down, the last Product will be 303570: This multiplied by .006101, or divided by 163.908, [vid. Table, page 55.] gives 1852.08.

Example. Let there be a Tun, whose Parallel Bases are Ellipses, the side of the greater Base be 108, the side of the lesser Base be 93, and the Depth be 30 Inches; all the same as in the last Problem; Therefore according to the Rule and Example there laid down, the last Product will be 303570: This multiplied by .006101, or divided by 163.908, [vid. Table, page 55.] gives 1852.08.

Example. Let there be a Tun, whose Parallel Bases are Ellipses, the side of the greater Base be 108, the side of the lesser Base be 93, and the Depth be 30 Inches; all the same as in the last Problem; Therefore according to the Rule and Example there laid down, the last Product will be 303570: This multiplied by .006101, or divided by 163.908, [vid. Table, page 55.] gives 1852.08.

Example. Let there be a Tun, whose Parallel Bases are Ellipses, the side of the greater Base be 108, the side of the lesser Base be 93, and the Depth be 30 Inches; all the same as in the last Problem; Therefore according to the Rule and Example there laid down, the last Product will be 303570: This multiplied by .006101, or divided by 163.908, [vid. Table, page 55.] gives 1852.08.

Problem VII.

Let Fig. 16. represent a Tun called a Prismoid, the Bases are Rectangular Parallelograms, unequal and disproportional, but parallel and alike situate; (a d) the Length below = 150 Inches, and (a b) the Breadth 100, (e h) the Length above 90, and (e f) the Breadth 70, and the depth 30. What is the Content in Ale-Gallons.

The R U L E.

1. **T**O the greater Length (a d) add $\frac{1}{2}$ the lesser Length (e h) and multiply the Sum by the greater Breadth (a b) reserving that Product.
2. To the lesser Length (e h) add $\frac{1}{2}$ the greater Length (a d) and multiply the Sum by the lesser Breadth (e f) add this Product to the former reserved Product.
3. Multiply the Sum of these two Products by $\frac{1}{3}$ of the Tun's Depth, and divide the last Product by 282, the Quotient will be the Content of the Tun in Ale-Gallons.

Example. (a d) 150 added to $\frac{1}{2}$ (e h) 45, the Sum is 195; this multiply'd by (a b) 100, the Product is 19500, which keep.

2. (e h) 90 added to $\frac{1}{2}$ (a d) 75, the Sum is 165; this multiplied be (e f) 70, the Product is 11550; this added to the former Product, viz. 19500, the Sum

Sect. VII. Of Gauging of Tuns. 79

Sum is 31050; this multiplied by $\frac{1}{3}$ of the Depth, viz. 10, the Product is 310500, and this divided by 282, gives 1101.06 the Content in Ale-Gallons.

The truth of this Rule will plainly appear, if we duly consider Fig. 13. in which $(e f c d)$ is equal to the greater Base, and $(e f g h)$ equal to the lesser Base of the Tun in the last Problem (which was represented by Fig. 16.)

In this, Figure 13, $(b i o v)$ is made equal to $(f e h g)$.

Also $(a b)$ 100 less by $(e f)$ 70, is equal $(a i)$

30. And $(a d)$ 150 less $(e b)$ 90, is equal to $(i d)$ 60, the Altitude is 30 as before.

The Lines being drawn, the whole Solid is composed of these parts, viz. three Prisms and a Pyramid.

The { 2 } Prism is { $i a s o b e$ }
 { $v c u p g$ }

The Pyramid is $o s n d h$.

The Content of these four Solids being added together, gives the Content of the whole Solid $a b c d e f g h$; for the whole is equal to all its parts taken together.

And the Content of these may be found thus:

1. Multiply $(f g)$ 90 by $(f e)$ 70, the Product is 6300; this multiplied by $(b f)$ the Altitude, viz. 30, gives 189000 the Content of the Prism, $b i o v f e h g$.

2. Multiply (*as*) 30 by (*as*) 90, the Product is 2700; this multiply'd by $\frac{2}{3}$ of the Altitude, *viz* 15, is 40500, the Content of the Prism *as before*.

3. Multiply (*en*) 70. by (*ne*) 60, the Product is 4200; this multiplied by 15, is 63000, the Content of the Prism *as before*.

4. Multiply (*sd*) 60 by (*so*) 30, the Product is 1800; this multiply'd by $\frac{2}{3}$ of the Altitude, *viz.* 10, gives 18000, the Content of the Pyramid *as before*.

So the Content of the	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right\}$	Prism is	$\left\{ \begin{array}{l} 189000 \\ 40500 \\ 63000 \end{array} \right\}$
Pyramid is	_____		18000
The Sum	_____		310500

Is the Content in square Inches, and is exactly the same with the Content found by the Rule given in the last Problem.

The Truth of this Rule being thus demonstrated, we shall shew its further Usefulness in the following Problems.

Pro-

Problem VIII. Figure 16.

There is a Tun (called a Cylandroid) whose Bases are both Ellipsical, unequal, and disproportionall (that is, there is not the same Proportion betwixt the Axis and Diameter of the greater Base, as is betwixt the Axis and Diameter of the lesser) let the given Dimensions be the same as in the last Problem, viz.

$$\begin{array}{l} n = 90 : p = 70 \\ i = 150 : m = 100 \end{array} \left| \begin{array}{l} \text{Rectangular} \text{ } \left\{ \begin{array}{l} \text{above} \\ \text{below} \end{array} \right. \end{array} \right.$$

And the Depth 30 Inches: What is the Content of this Tun in Ale Gallons?

For the Content of this Tun, work by the General Rule, and according to the Example given in the last Problem, only instead of 282, divide the last Product by 359.05. So in this Example (the Numbers being the same as in the last) the last Product will be 310500, this divided by 359.05 quotes 864.78, the Content in Ale Gallons.

Problem IX. Fig. 15.

There is a Tun whose Bases are Rectangular Parallelograms, and Parallel, but neither equal nor alike situate (for gh , the Breadth above, is posited directly opposite to ad , the Length below) how many Ale-Gallons will this Tun contain?

The Dimensions are,

bg Breadth = 68: he Length = 85 above.

da Length = 100: dc Breadth = 80 Below.

The Depth 30 Inches.

If the Bases of this Tun were alike situate (as is supposed in Problem II. of this Section) the Content would be 729.76 Ale-Gallons, as was there shewn; but the Bases being inverted as abovesaid, the Content will be found to be 735.81 Ale-Gallons, as appears by the following Work, which is performed by the General Rule laid down in Problem VII. of this Section.

Thus (bg) 68 added to $\frac{1}{2} (da)$ 50, is 118; this multiplied by (he) 85, the Product is 10030.

Again (da) 100 added to $\frac{1}{2} (bg)$ 34, is 134; this multiplied by (dc) 80, the Product is 10720, the Sum of these two Products is 20750; which multiply'd by $\frac{1}{3}$ of the Depth, viz. 10, gives 207500, the Content in square Inches; this divided by 282, the Quotient is 735.81 the Content in Ale-Gallons. Or if the Sum of the last Products be divided by 359.05 the Quotient will be 577.91 the Content of an Elliptical.

Sect. VII. Of Gauging of Tuns.

82

tical Tun, whose Axis above is directly opposite to the shorter Diameter below; and such an Elliptical Tun may be conceived to be inscribed in the other, the respective Axis's and Diameters, Lengths, and Breadths, and also the Depth being the same in both.

Problem X. Fig. 14.

To find the Content of a Tun whose Bases are unlike, suppose one Square, and the other Oblong; or one Round, and the other Elliptical, but Parallel one to the other, and the Sides of the Tun straight.

The Dimensions are,

Breadth.

Length.

$e f = 74$; $f g = 100$ } Rectangular { above.

$d e = 100$; $c b = 100$ } Conjugates { below.

Depth 30 inches.

By the general Rule in Problem VII. work thus:

1. To $(e f)$ 74 add $\frac{1}{2}$ $(d e)$ 50, the Sum is 124, this multiplied by $f g$ 100, is 12400.

2. To $(d e)$ 100 add $\frac{1}{2}$ $(e f)$ 37, the Sum is 137, this multiplied by $(c b)$ 100 is 13700; this added to the former Product, viz. 12400, makes 26100, and this multiplied by $\frac{1}{3}$ of the Depth, viz. 10, the Product is 261000; this divided by 282, gives 925.53, the Content of this Tun in Ale-Gallons, supposing the Bases are one Square, and the other Oblong as the Figure represents them.

But if the Bases were one Round, and the other Elliptical, the last Product, viz. 261000 must be

divi-

84 *Of Inching of Tuns.* **SECT. VIII.**
divided by 359.05, and in this case the Content
will be 726.91.

By what hath been said, I presume it will not be
difficult to find the whole Content of any Tun
whose Bases are parallel, and Sides streight from
Top to Bottom.

S E C T. VIII.

*Shewing how to find the Content of any Tun
from Inch to Inch, or what any Tun will con-
tain upon every Inch of its Depth, which is
commonly call'd Inching of a Tun.*

Problem. I.

*Suppose a Tun whose Bases are both Round, or
both Square and Parallel one to another, and
in the Sides a streight Line may be every
where apply'd from Base to Base: Let the
Diameter, or Side of the greater Base be
187.2 Inches, the Diameter or Side of the
lesser Base 180, and the Depth 72 Inches, to
Inch this Tun (be it Round or Square) from
the lesser Base downward.*

R U L E.

FROM the Side or Diameter of the greater, sub-
tract the Side or Diameter of the lesser Base, di-
vide the Difference by the Tun's Depth, and call the
Quotient (*s*). For

Sect. VIII. Of Loading of Tuns.

858

For the Side or Diameter of the lesser Base, put (b).

Multiply the Number (b) by the Number (a) and call the Product (c). Then,

1. To the Square of (b) add the Number (c), and one third part of the square of (a), the Sum of these three will shew the Content of the first Inch of the Tun.

2. Multiply one third part of the Square of (a) by 2; to this Product add the Number (c); then multiply the Sum by 2, and to this Product add the Square of (b); multiply this Sum by 2, this last Product is the Content of the two first Inches of the Tun.

3. Multiply one third part of the square of (a) by 3; then add and multiply as in the last, the third and last Product will shew the Content of the three first Inches of the Tun.

4. Having got these three Numbers, subtract the first from the second, and likewise the second from the third; then the first Number, and each of these Differences or Remainders being severally divided by 282 square Tuns, or by 359.05 for round Tuns, the respective Quotients will be the Content (in Ale-Gallons) of the first, second and third Inches of the Tun, by which a Table may be made for the whole Tun, as in the Example following.

Example of a Square Tun.

The Side of the great Base ————— 187.2
The Side of the lesser Base ————— 180.0

The Remainder is ————— 7.2

This divided by the Tun's Depth, viz. 12. quotes
6 for the Number (a).

The

The Number b (which is the side of the lesser Base) is 180; this multiply'd by the Number a viz. 6, gives 108 for the Number c .

1. The Square of b is _____ 32400
 Number c is _____ 108
 $\frac{1}{3}$ of the Square of a is _____ .12

The Sum is the Content of the first
 Inch of the Tun, in Inches, viz. } 32508.12

2. $\frac{1}{3}$ of the Square of a is _____ .12
 Multiply by _____

Product _____ .14
 Add the Number c _____ 108.00

Sum is _____ 108.14
 Multiply by _____ 2

Product _____ 216.48
 Add the Square of the Number b _____ 32400.00

Sum is _____ 32616.48
 Multiply by _____ 2

Product is the Content of the two
 first Inches of the Tun, in Inches, } 65232.96
 viz.

The Side of the great Base
 The Side of the lesser Base
 The Remainder is
 This divided by the Tun's Depth, viz. 12
 gives for the Number d 5436.08

3. $\frac{1}{3}$ of the Square of _____ .12
Multiply by _____ 3

Multiply by _____ 3

Product _____ :36

Product	_____	.36
Add the Number	_____	108.00

Sum is ~~108.36~~ 108.36

Multiply by ~~$\frac{1}{\sqrt{2}}$~~ $\frac{\sqrt{2}}{2}$ ~~$\frac{1}{\sqrt{2}}$~~ $\frac{\sqrt{2}}{2}$

~~Product of China~~ 325.08

Product	325.08
Add the Square	324.00

Sum is 32725.08

Multiply by _____ and 3

Product is the Content of the
three first inches of the Tun, ————— 98173.24

three first inches of the Tun, $\frac{1}{2}$ — 98175.24

in Inches, and

Two first Inches ----- 64232.06

First Inch ————— 32508.12

Ref for second Inch ~~_____~~ 33734-84

Three first Inches. 08175.24

Two first inches ~~69232.96~~

Rest for third Inch _____ 13942.38

Now these three Numbers, 2 015 10 4 2

viz. $\left. \begin{array}{l} 32508.12 \\ 32724.84 \\ 32942.28 \end{array} \right\} \begin{array}{l} \text{Divided} \\ \text{by } 282 \\ \text{quotes} \end{array} \left. \begin{array}{l} 115.277 \\ 116.045 \\ 116.816 \end{array} \right\}$

For the Content (in Ale-Gallons) upon the first, second, and third Inches of this Tun, supposing the Bases square.

And if the said Numbers be divided by 359.05, the respective Quotients will

be $\left. \begin{array}{l} 90.939 \\ 91.742 \\ 91.747 \end{array} \right\}$ the Content of the first, second and third Inches of the Tun, when its Bases are Round.

Note, by these Numbers the following Table is composed for the Round Tun. Thus,

Take the first Inch from the second, the Remainder or Difference is .603, which set down in the third Column, against the space betwixt the first and second Inches. In like manner take the second Inch from the third, the Remainder is .605, which place in the third Column under the other; then take .603 from .605, there remains .002 for the second Difference, which is always the same, as in the fourth Column, this second Difference added to the second Number in the third Column viz. .605, makes .607, and this added to the Content of the third inch, viz. 91.747, makes 92.354, the Content of the fourth Inch; and so by a continual Addition of the second Difference to the first, and of their Sum to the Inch last found, the Table is made.

Depth

Depth. Inches.	Content of every Inch in Ale Gall.	First Differ.	Second Differ.
1	90.539		
2	91.142	.603	.002
3	91.747	.605	.002
4	92.354	.607	.002
5	92.963	.609	.002
6	93.574	.611	.002
7	94.187	.613	.002
8	94.802	.615	.002
9	95.419	.617	.002
10	96.038	.619	.002
11	96.659	.621	.002
12	97.282	.623	

Sum--1126.706--Content of the Tun.

In this Example we have made a Table for the Round Tun, and the Operation would have been the same for the Square Tun, had we taken the three first Inches of *this* instead of *these*.

Note, When we Inch a Tun by this Rule, we suppose it to be Regular, having all the Qualifications exprest in the Problem. But forasmuch as most Tuns have some irregularity or Unevenness in their Sides, I shall here lay down a more Practical Rule for inching of Tuns.

Pre-

Problem II.

There is a Tun whose Diameter at the Bottom is 108 Inches, the Diameter at the Top 93, and the Depth 30; how many Ale-Gallons will this Tun contain upon every Inch of the Depth?

TO resolve this Question, there must first be known the whole Content of the Tun, and also the Area of the mean Diameter in the middle of every 10 or 12 Inches of the Depth, and these may be thus found:

Take (with some convenient Instrument) a Diameter in the middle of every 10 Inches of the Tun's Depth, and suppose in this Example the first (at 5 Inches from the Top) be 95.5, the second (at 15 Inches from the Top) 100.5, and the third 105.5; these I set down as in the following Table:

This done, we may consider the Tun either as Round or Square, and first to Inch it as a Round.

Set the Gage-Point (a g) to 1; then against the several Diameters already found, you have their respective Area's: So the first Area will be 25.40, the second 28.13, the third 31.00; these I set down in the Table against their Diameter, and having added them together, the Sum is 84.53, this multiplied by 10 (or which is all one, remove the Prick one place toward the Right-hand) the Product is 845.3, the whole Content of the Tun.

Note,

SECT. VIII. Of Inching of Tuns.

91

Note, Here the Diameters being taken in the middle of every 10 Inches, there will be no use of Multiplication or Division in Inching the Tun, as there will be when the Diameters are taken in the middle of every Foot or half Foot; for here the Areas of the several Diameters do not only shew the mean Inch in the middle of every 10 Inches, but also the whole Content of those 10 Inches if you remove the Prick but one place more towards the Right-hand. Thus the mean Inch of the last 10 Inches is 31.00, but if the Prick be removed as abovesaid, it is 316, which is the Content of a Tun at 10 Inches deep; I therefore reduce the several Contents in the third Column into Barrels, Firkins, and Gallons, Beer-measure, and set them down in the fourth Column, these added together are 123 Barrels, 11 Firkin, 8 Gallons, and 3 tenths of a Gallon, which agrees with the Sum of the Numbers in the third Column.

Parts of the Tun's Depth.	Mean Diameter in the middle of every 10 Inches of the Tun's Depth.	Mean Area or Contents of every 10 Inches of the Tun's Depth.	Content of every 10 Inches Beer measure.
10	95.5	25.40	7 0 2.0
10	100.5	28.13	7 3 2.3
10	105.5	31.00	8 2 4.
30		84.53	13 10 8.3

By these Numbers thus found it will be easie to make a Table, which shall shew the quantity of Liquor contain'd in the Tun at any depth, for which observe the following Directions.

1. Set

1. Set down the whole Content of the Tun in Barrels, Firkins, Gallons, and Parts, in their respective Columns, and at the top of the first Column against the Content set 0, and under it the Tun's Depth in Inches, 1, 2, 3, 4, 5, &c. this done take the first mean Inch, viz. 25.4, and reduce it into Barrels, Firkins, and Gallons, Beer-Measure, it makes 0 : 2 : 7 : 5 ; set this upon a scroll of Paper, and substract it continually from the whole Content till you come at the tenth Inch of the Tun's depth, which done, if you mistake not, the Remainder there will be 16 : 1 : 6 : 3 :

2. Take the second mean Inch, viz. 28, 13, this being reduced as the former is 0 : 3 : 1.13. substract this continually from the last Remainder, till you come at 20 Inches deep, and there your Remainder will be 8, 2, 4.

Take the third and last mean Inch, and reduce and substract it as before, and if your Work be right, there will be no remainder at the last Inch or Bottom of the Tun ; now the Table being made, you may cast away the Fractions as useless. The use of this Table is obvious to the meanest Capacity, without any further Explanation :

In.	P.	P.	G.
0	13	1	8.30
1	22	3	9.9
2	22	0	3.5
3	21	1	4.1
4	20	2	5.7
5	19	3	7.3
6	19	0	8.9
7	18	2	1.5
8	17	3	3.1
9	17	0	4.7
10	16	1	6.3
11	15	2	7.9
12	14	3	9.5
13	14	0	1.1
14	13	1	2.7
15	12	2	4.3
16	11	3	5.9
17	10	4	7.5
18	10	0	9.1
19	9	1	1.7
20	8	2	3.3
21	7	3	4.9
22	6	4	6.5
23	6	0	8.1
24	5	1	9.7
25	4	2	1.3
26	3	3	2.9
27	2	4	4.5
28	1	5	6.1
29	0	6	7.7
30	0	0	9.3

Sect. VIII. Of *Inching of Tuns.*

93

For if you come to this Tun, and find 2 inches of the Depth dry, the Quantity of Liquor then in the Tun is 22 B. 0 F. 3 G. If 7 inches be dry, there remain in the Tun but 18, 2. 1, &c. Thus for Round Tuns.

Now, suppose the Tun last mention'd had been Square, and the Sides at Bottom and Top equal to the Diameters of the last, and the Depth the same; the Diameters or Sides in the middle of every 10 inches of the Tun's depth will be the same as in the last; by which the several mean inches, and also the whole Contents of every 10 inches are found to be as in this Table.

Parts Tun's Depth.	Sides of the Tun in the middle of every 10 Inches.	Mean A- rea's or Contents of every ten Inches.	Content of every 10 Inches Beer-Measure.		
			B.	F.	G.
10	95.5	32.34	8	3	8.4
10	100.5	53.81	9	3	7.1
10	105.5	39.46	10	3	7.6
30			29	3	5.11

For, set 282 upon B, to 95.5 the first Diameter upon A, then against 95.5 upon B, is 32.34 the first mean inch, and after the same manner find the second and third, which being reduced as in the last Example, I find the whole Content to be 1076.1 Ale-Gallons (or which is the same thing) 29 B. 3 F. 5.1 G. this being done, you may proceed to inch this Tun by the same directions which were given in the last Example.

Pro-

Problem III.

To Inch a Tun whose Bases are unlike, viz. the lower Circular, and the upper Elliptical, suppose the Diameter of the Circular Base be 100 Inches, the Axis (or longest Diameter) of the Elliptical Base 100, and the shortest 90, and the Tun's Depth 20 Inches.

TO Inch this Tun, I take the cross Diameters to the middle of every 4 Inches of the Tun's depth: Thus at 2 Inches from the Top I find the longest Diameter 100, and the shortest 91, in the middle of the next four Inches (which is 6 Inches from the Top) the longest Diameter is 100, and the shortest 93, and thus I find the rest, which are as in this Table is express'd: Now having these Diameters, the Area or mean Inch in the middle of every 4 Inches of the Tun's Depth may be thus found.

Set 359.05 upon B, to the longest Diameter, (viz. 100) upon A, then against the shorter Diameter (viz. 91) upon B, is 25.34, the first mean Inch, or Area in Ale-Gallons; and after the same manner the second and third will be found, as in the following Table.

mean inch
first mean inch
second mean inch
third mean inch
fourth mean inch
fifth mean inch
sixth mean inch
seventh mean inch
eighth mean inch
ninth mean inch
tenth mean inch
eleventh mean inch
twelfth mean inch
thirteenth mean inch
fourteenth mean inch
fifteenth mean inch
sixteenth mean inch
seventeenth mean inch
eighteenth mean inch
nineteenth mean inch
twentieth mean inch

100

Depth

Depth.	Anis.	Shor- ter Dia.	Mean Inch in Ale- Gallons.	Contents of every 4 Inches of the Tun's Depth.
4	100	91	25.34	2 3 2.36
4	100	93	25.88	2 3 4.52
4	100	95	26.47	2 3 6.88
4	100	97	27.02	3 0 0.08
4	100	99	27.57	3 0 2.28
0			132.28	14 2 7.12

Now if the Mean Inches be severally multiplied by 4, and these Products reduced into Barrels, Firkins, and Gallons, Beer-measure, they will give the Numbers in the last Column; which added together are 14 Barrels, 2 Firkins, 7 Gallons, and 12 parts of a Gallon.

Set this down to the first Number of your Table, and then reduce the first mean Inch into Barrels, Firkins, and Gallons, and subtract it 4 times from the whole Content and Remainders; do the like with the other mean Inches, and so compleat the Table as in the last Example.

Problem I.

To find the Drip or Fall of a Tun, and to make Allowance for the same in Inching the Tun.

Example.

Admit the Tun last mentioned were so placed, that when the Bottom is but just covered on one side, the Liquor is 4 Inches deep on the side

opposite; How much must be allow'd for the Fall of this Tun.

By the Operation foregoing the Area of the mean Diameter in the middle of the 4 last Inches (that is, at 2 Inches from the bottom) was found to be 27.57, this multiplied by 4, is 110.28 Ale-Gallons, the half of this is 55.14, and so much may be allow'd for the Fall of the Tun; reduce these 55.14 Gallons into Barrels, Firkins, and Gallons Beer-measure, gives 1 : 2 : 1.14 : and this taken from 24 : 2 : 7.12, the Content of the Tun when it stands level, there remains 13 Barrels, 0 Firkins, 5 Gallons, and 98 parts: And so much will the Tun contain, when the Fall is 4 inches; therefore set this at the Top of the Table, then reduce the first mean Inch viz. 25.34 into Barrels, Firkins, and Gallons, and subtract it four times from the whole Content, 13 : 0 : 5 : 9.8 the remainder will be 10 : 1 : 3 : 62 : Do the like with second, third, and fourth mean inches, and you will find the Remainder at 16 inches dry, to be 1 : 1.14 : And so much it takes to cover the Bottom, as was before observed.

As for such Tuns as are supposed to be the Frustums of *Spheroids*, *Parabolick Conoids*, *Hyperbolick Conoids*, or *Parabolick Spindles*, I shall give Rules for Gauging, and inching of these in the Appendix, where I shall also endeavour to explain the nature of these Figures, and shew whence they are derived.

0	13	05.98
1	12	17.64
2	11	30.30
3	10	01.96
4	9	13.62
5	8	24.74
6	7	35.86
7	6	06.98
8	5	18.10
9	4	28.63
10	3	00.16
11	2	10.69
12	1	21.22
13	0	31.70
14	0	41.78
15	0	51.16
16	0	01.14

Problem V. Figure 20.

To Gauge a Copper, and make Allowance for the Crown; let a , b , c , d , e , a , represent a Copper whose Content is required.

1. **T**O find the several Diameters and Depth of the Copper, together with the Altitude of the Crown,

Take a small Cord, or Thread, make one end fast at a , and extend the other to the opposite Side of the Copper at e , where make it fast, or cause some Person to hold it very straight; then set one End of the Instrument in the Bottom of the Copper at b , and move it to and fro, till you find the nearest Distance to the Thread (as at c), this Distance ($b c$) is the Depth of the Copper, suppose it be 44 Inches.

In like manner set the End of the Rule upon the Top of the Crown at e , and take the nearest Distance to the Thread (as $c v$), suppose it be 36 Inches, this subtracted from ($b c$) 44, the Remainder 8, is the Altitude of the Crown.

To find ($b d$) the Diameter of the Bottom of the Crown,

Measure ($a e$) the Diameter at the Top, admit it be 115 Inches, then hold a Thread so as a Plummer at the End thereof may hang just over b ; by this means you may find the Distance $a o$, and on the other Side $e s$, suppose each be 14.5, add these together, and subtract their Sum (*viz.* 29) from $a e$, 115, the Remainder 86, is equal to $b d$, the Diameter at the Bottom of the Crown: The Diameter which touches the Top of the Crown may

F

be

be found by the Instrument, suppose it be 91.4 Inches.

Now, to find the Content of the Copper from the Crown upwards (that is, the part *abcie*) the Depth (*vc*) being 36 Inches, you may take a Diameter in the middle of every 6 Inches betwixt *v* and *c*, which suppose to be as in the second Column of the following Table, the Numbers in the third Column are the respective Area's in Ale-Gallons, found by the *Gage-Point* as before; the fourth Column shews the Content of every 6 Inches in Barrels, Firkins, and Gallons Beer-measure: These added together are 29 Barrels, 3 Firkins, 2.5 Gallons.

And so much will the Copper contain after the Crown is cover'd.

Now if the Crown be taken for the Frustum of a Sphere, the Content (by the third Problem of *Section V.*) will be found to be 83.3 Gallons.

But the Content may be more readily found, very near the truth: Thus,

The Diameter *bd* was found to be 86, the Area to this Diameter is 20.6, this Multiplied by $\frac{2}{3}$ the Crown's Altitude, *viz.* 4, gives 82.4, the Content of the Crown, this Subtracted from the Content of the part or portion *bcidnb*, the Remainder is the quantity of Liquor that will cover the Crown; the Content of the part *bcidnb* may be thus found, *bi* is 91.4, and *bd* is 86, these added together are 177.4, the half 88.7 may be taken for a mean Diameter, the Area to this Diameter is 21.9, this Multiplied by 8, the Altitude, gives 175.2, the Content.

Now Subtract 82.4 (the Content of the Crown) from 175.2, the Remainder is 92.8, or 2 Bar. 2 Fir. 2 Gal. and .8 of a Gal. this added to the Content

first

Sect. VIII. Of Gauging of Coppers. 99

first found. viz 29 Bar. 3 F. 1.5 Gal. gives 32 B. 1 F. 5.3 Gal. the Content of the whole Copper, as in the following Table.

Parts of the depth.	Diameter.	Mean Inch	Content of every 6 Inches in B. F. G.		
6	113.1	35.62	5	3	6.7
6	109.2	33.21	5	2	1.2
6	105.28	30.86	5	0	5.1
6	101.36	28.61	4	3	0.6
6	97.33	26.37	4	1	5.2
6	93.4	24.29	4	0	1.7
36	Sum —————		29	3	2.5
To cover the Crown —————			2	2	2.8
Content Copp —————			32	1	5.3

The Content being thus found, you may proceed to Inch the Copper by the same Directions which were given for Inching of Tuns in the last Problem.

Problem VI.

The Diameter of any Round, or Side of any Square Mash-Tun, being given, to find how many Gallons of Mault it will contain at one Inch deep.

THE Corn-Gallon (which is the Measure we here account by) is supposed to contain 268.8 Cubick inches.

F 2

But

100 *Of Gauging of Mash-Tuns. Sect. VIII.*

But Mr. Dary in his Compleat Gauger, saith, that an indifferent sort of Mault, and of an indifferent Grinding, when three Worts shall have passed through it, will be more compact by about a sixth part; he therefore concludes, that 227 may be a proper Divisor for Square Mash-Tuns, and 388 for Round ones.

1. *For Round Mash-Tuns.*

The Diameter of that Circle, whose Area is 227, is 17 and this may be called the *Gage-Point* for Round Mash-Tuns; and to find the Content of any such Mash-Tun,

Set 17 upon D, to 1 upon C; then against any Diameter upon D, you have the Content in Gallons at one inch deep: Thus if the Diameter be 113, the Area or Content at one inch deep would be 44.3 that is, 5 Bushels, and 4.3 Gallons.

2. *For Square Mash-Tuns.*

If a Mash-Tun be Square or Oblong, the Content at one inch deep may be thus found: Say,

As 227 is to the Length;

So is the Breadth to the Content.

Example. Suppose the Length 112 inches, and the Breadth 80, how many Gallons will this Tun contain at one inch deep? Answer, 39.4: For,

Set 227 upon A, to 80 upon B; then against 112 upon A, is 39.4, the Content upon B.

SECT.

S E C T. IX.

Of Cask-Gauging.

BEFORE we can find the Content of a Cask, we must know the Dimensions of it, viz.

The { Bung } Diameter, } in inches.
 { Head }
 { Length, }

And when these are known, the form or shape of the Cask must be consider'd; for although the Diameters (abovesaid) and the length of one Cask may be equal to those of another, yet one of these Casks may contain several Gallons more than the other, and therefore the Content of all Casks cannot be found by one and the same Rule.

Now most Writers in Treating of this Subject, have taken it for granted, that every common Cask (as Butt, Pipe, Hoghead, Barrel, and such like,) is in the form of one of these Solids, viz.

1. The middle Frustum of a Spheroid.
2. The middle Frustum of a Parabolical Spindle.
3. The middle Frustum of two Parabolical Conoids, abutting upon one common Base; or,
4. The middle Frustum of two Cones, abutting upon one common Base.

Accordingly they have laid down Rules for finding the Content of these Solids, and by those

Rules they suppose the Content of any Cask may be found.

As for instance.

If the Dimensions of a Cask be

Bung } Diameter } $\begin{matrix} 32 \\ 24 \\ 40 \end{matrix}$ } Inches.
Head }
Length —————

By the Rule given by Mr. Dary, and others, the Content of this Cask if taken for

			Ale-Gall.	
The middle Frustum of	{	a { Spheroid —————	} is {	97.455
		Parabolical Spindle.		96.505
		b { Parabolic. Conoids		89.136
		Cones —————		87.934

The Rules are true, as to the first and last of these Solids, viz.

The middle Frustum of a Spheroid,
two Cones.

And these may be represented by Figure 21.

A Cask which is the middle Frustum of a Spheroid is shewn by the outermost Line $d, o, a, d, f,$ $c, o, b, o, c, e, d,$ the two straight Lines $d, e, c,$ and $d, f, e,$ are the Heads of the Cask, and the Curved Lines $d, o, a, o, d,$ and $c, o, b, o, c,$ the Staves.

If

If a Cask (can) be in the form of the middle Frustum of two Cones, it may be represented by the straight Lines d, u, a, u, d , and c, u, b, u, c , the Heads d, c, c , and d, f, c , the Bung-Diameter a, b , and the Length e, f , are all the same in both these Casks, and yet 'tis evident by the Figure it self, that one will contain more than the other, and by the Example above cited, the difference is 9.521, Gallons, for the Spheroid contains 97.455 Gallons, and the Frustum of two Cones but 87.934.

There are very few (if any) Casks whose Diameters and Length are equal to those abovenamed, that will contain more than 97.455 Gallons (the Contents of the Spheroid) nor less than 87.934 (the Content of the Frustum of two Cones) so that all the varieties that can happen will fall betwixt these two; and the Staves of any Cask differing from these, will lie between the Lines d, o, a , and d, u, a , in Figure 21.

Now for the other two Solids, viz. the middle Frustum of a *Parabolical Spindle*, and the middle Frustum of two *Parabolical Conoids*; according to the Authors above-cited, the Content of the first is 96.505 Gallons, and the second 89.136.

Whether the Rules they have given for finding these Contents be true or false, I shall examine in the Appendix.

But supposing they are true, yet they do not answer the various cases that may happen in common Practice; for in the Example above, the Content of a Cask taken, as the middle Frustum of a *Parabolical Spindle*, is but .95 of a Gallon less than the Content of the Frustum of a *Spheroid*, and the Content of the Frustum of two *Parabolical Conoids* is but 1.931 more than the Frustum of two Cones.

Now I presume no Man will pretend to distinguish which Notion a Cask must be taken in, whether as the Frustum of a *Spheroid*, or as the Frustum of a *Parabolical Spindle*, when the Contents differ not so much as a Gallon.

Moreover, here is above 7 Gallons Difference betwixt the Contents of the Frustum of a *Parabolical Spindle*, and the Frustum of two *Parabolical Conoids*: So that I may modestly affirm, that there are many Casks whose Content cannot be found by any of these Rules; for it must be granted, that a Cask whose Diameters and Length are equal to those in this Example, may contain some certain quantity betwixt 96 Gallons, and 89.

As suppose 93, I say, if this Cask were to be Gauged by any of the Rules above-cited, it will give the Content at least 3 Gallons too much, or 4 Gallons too little.

This premised, I come now to lay down some short Rules, by which the Content of any Cask may be found very near the Truth.

Problem. I. Fig 21.

There is a Cask supposed to be the middle Frustum of a Spheroid, intercepted between two Plains Parallel, cutting the Axis at Right Angles, let ab the Diameter at the Bung be 32 Inches, dc the Diameter at the Head 24, and the Length ef 40, Inches; How many Ale or Wine-Gallons will this Cask contain?

R U L E.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add half the Difference of the said Squares; the Sum of these multiplied by the Length, and this Product divided

By $\left| \begin{array}{l} 1077 \\ 882 \end{array} \right|$ Quotes the $\left| \begin{array}{l} \text{Ale} \\ \text{Content in} \end{array} \right|$ $\left| \begin{array}{l} \text{Wine} \\ \text{Gallons.} \end{array} \right|$

Example. $\left| \begin{array}{l} \text{Wine} \\ \text{Gallons.} \end{array} \right|$

Square of ab 32 is 1024

Square of dc 24 is 576

Sum is 1600

Half Sum 800

Half the Difference of the Square is 224

Sum is 2624

$\text{F } 5$

Which

Which multiplied by 40 (the Length of the Cask, gives 104960, this Divided by 1077, Quotes 97.455 the Content in Ale-Gallons.

Or divided by 882, the Quotient will be 119.002, the Content in Wine-Gallons.

Note, The Common Rule for Gauging the Frustum of a Spheroid, (by taking twice the Square of the Bung-Diameter, and the Square of the Head-Diameter, &c.) is somewhat a shorter way than this: But this Rule being more agreeable to the Work in the following Problems, I have thought fit to insert it here: The Reader may use which he pleases.

Problem II.

Suppose a Cask (of the same Diameters, and Length with the former) shall have its Staves betwixt the Bung and Head less Curved, so as to leave out about a third part of the space between the Lines $d o a$, (which represent the Spheroid) and $d u a$, as in Figure 22. How many Gallons will this Cask contain?

R U L E.

TO the Sum and half Sum of the Squares of the Bung and Head-Diameters, add three tenth parts of the Difference of the said Squares; the Sum of these Multiplied by the Length, and the last Product divided by 1077, or 882, Quotes the Content in Ale or Wine-Gallons respectively.

Example.

Example. *The Dimensions as above.*

Sum of the Squares is ————— 1600

Half Sum is ————— 800

The Difference of the Squares Multiplied by 3, is ————— 1344

Sum of these is ————— 25344

This multiplied by 40, gives 101376, and this Divided by 1077 quotes 94.128, the Content in Ale-Gallons; or by 882, the Quotient will be 114.938, the Content in Wine-Gallons.

Problem III. Figure 19.

There is a Cask whose Staves, between the Bung and Head, have but little Curvature, lying nearer to the straight Line and, than to the Curved Line and, What is the Content of this Cask in Ale or Wine-Gallons?

R U L E.

TO the Sum and half Sum of the Squares of the Bung and Head-Diameters, add one tenth of the Difference of the Squares; the Sum of these Multiplied by the Length, and this Product divided as above directed, quotes the Content in Ale or Wine-Gallons respectively.

Example.

Example. (*Dimensions as above.*)

Sum of the Square is ————— 1600

Half Sum is ————— 800

One tenth of the Difference is ————— 44.8

Sum of these is ————— 2444.8

Which Multiplied by 40, gives 97792, this Divided by 1077, the Quotient is 90.800, the Content in Ale-Gallons; or by 882, quotes 110.875, the Content in Wine-Gallons.

Problem IV. Figure 21.

To find the Content of a Cask in form of the middle Frustum of two Cones abutting upon one common Base (if any such can be) it may be represented by the strait Lines *dua*, and.

R U L E.

From the Sum and half Sum of the Squares of the Bung and Head Diameters, Subtract half the Square of the Difference of the Diameters themselves; Multiply the Remainder by the Length, and Divide this Product as in the last, the Quotient will be the Content in Ale or Wine-Gallons.

Example.

Sect. IX. Of Cask Gauging.

109

Example. (Dimensions as above.)

Sum of the Squares is 1600

Half Sum is 800

Sum of these is 2400

Difference of the Diameter is 8, this } 32

Squared is 64, the half is } 32

The Remainder is 2368

This Multiplied by 40, gives 94720, and this Divided by 1077 quotes 87.948, the Content in Ale-Gallons; or by 882 the Quotient will be 107.392 the Content in Wine-Gallons.

Thus it appears by the four Problems foregoing, if the Dimensions of a Cask

Be { Bung } Diameter { 32 }
 { Head } { 24 } Inches,
 { Length } { 40 }

The Content in each variety is as followeth;

	Ale-Gall.	Diff.
By Problem { I.	97.455	
II.	94.128	3.327
III.	90.800	3.328
IV.	87.948	2.852

And

And here it may be observed, that the Differences of the Contents thus found are very near equal : So that by these Rules there is not so much room for Error, as by the Rules which several Authors have prescribed.

I have insisted the longer upon this Business, that I might prevent Mistakes, for we cannot be too exact, where a small Error must needs be prejudicial either to the King, or to the Merchant, especially when the Duty upon Brandy is great.

Problem V.

The Dimension of any Cask being given to find the Content by the Instrument.

TO do this, we must first find a mean Diameter, that is, such a Diameter as shall reduce the Cask to a Cylinder ; for which purpose I have Calculated these three Tables, for the three first varieties of Casks abovementioned.

A TABLE for Equating the Diameters of the middle Frustum of a Spheroid.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.67	0.73	0.80	0.87	0.94	1.01	1.08	1.15	1.22	1.29
2	1.36	1.43	1.50	1.57	1.64	1.71	1.78	1.85	1.92	1.99
3	2.06	2.13	2.20	2.27	2.34	2.41	2.48	2.55	2.62	2.69
4	2.76	2.83	2.90	2.97	3.04	3.11	3.18	3.25	3.32	3.39
5	3.46	3.54	3.61	3.68	3.75	3.82	3.89	3.96	4.03	4.10
6	4.17	4.24	4.31	4.39	4.46	4.53	4.60	4.67	4.74	4.81
7	4.88	4.95	5.03	5.10	5.17	5.25	5.32	5.39	5.46	5.53
8	5.60	5.68	5.75	5.82	5.90	5.97	6.04	6.11	6.18	6.25
9	6.32	6.40	6.47	6.55	6.62	6.70	6.77	6.84	6.91	6.98
10	7.05	7.13	7.20	7.28	7.35	7.47	7.49	7.56	7.64	7.71

A TABLE for the Second Variety of Casks.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.62	0.66	0.74	0.81	0.87	0.93	0.99	1.06	1.12	1.18
2	1.24	1.30	1.36	1.43	1.49	1.55	1.62	1.68	1.74	1.80
3	1.86	1.93	1.99	2.05	2.12	2.18	2.24	2.30	2.37	2.43
4	2.49	2.55	2.62	2.68	2.75	2.81	2.88	2.94	3.01	3.07
5	3.13	3.20	3.26	3.32	3.39	3.46	3.52	3.59	3.65	3.72
6	3.78	3.85	3.91	3.98	4.04	4.11	4.18	4.24	4.31	4.37
7	4.44	4.50	4.57	4.63	4.70	4.76	4.83	4.90	4.96	5.03
8	5.10	5.16	5.23	5.30	5.36	5.43	5.49	5.56	5.63	5.70
9	5.77	5.84	5.91	5.97	6.03	6.10	6.17	6.24	6.30	6.37
10	6.44	6.51	6.57	6.64	6.71	6.77	6.83	6.90	6.96	7.03

A TABLE for the Third Variety of Casks

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.55	0.66	0.65	0.70	0.76	0.81	0.87	0.93	0.99	1.05
2	1.11	1.17	1.22	1.28	1.33	1.39	1.44	1.50	1.55	1.61
3	1.67	1.73	1.78	1.84	1.90	1.95	2.01	2.06	2.12	2.18
4	2.24	2.30	2.35	2.41	2.46	2.52	2.57	2.63	2.69	2.75
5	2.81	2.87	2.93	2.98	3.04	3.10	3.16	3.21	3.27	3.33
6	3.39	3.45	3.51	3.56	3.62	3.68	3.73	3.80	3.85	3.91
7	3.97	4.02	4.08	4.14	4.20	4.26	4.32	4.38	4.44	4.50
8	4.56	4.62	4.68	4.74	4.80	4.86	4.92	4.98	5.04	5.10
9	5.16	5.22	5.28	5.33	5.41	5.47	5.53	5.59	5.65	5.71
10	5.77	5.83	5.89	5.96	6.02	6.08	6.14	6.20	6.26	6.32

Now by these Tables the mean Diameter of any Cask is thus found.

Subtract the Head-Diameter from the Bung-Diameter, then seek the Difference in the Table, taking the whole inches in the first Column, and the Tenths (if any be) at the Head, in the Angle of meeting you have a Number, which being added to the Head-Diameter, gives the mean Diameter sought.

Example,

Example. Let the Dimensions be as in the first Problem of this Section, the Difference of the Diameter is 8. In the Table for a *Spheroid*, against 8 on the Side, and under 0, I find 5.6; this added to the Head-Diameter (*viz.* 24.) the Sum is 29.6, the mean Diameter sought.

This being obtained, the Content of any Cask may be found by the Instrument at one Operation, for which the Proportion is:

*As the Gage-Point is to the Cask's Length;
So is the mean Diameter to the Content.*

Example. The Length 40, and the mean Diameter 29.6 Inches.

Set the Gage-Point for Ale-Gallons to 40 upon C, then against 29.6 upon D, is 97.455, the Content in Ale-Gallons, which is the same as was found by the first Problem of this Section.

Note. The three Tables above are expressed in three Scales, and placed upon one Side of the Rule against the Line of Inches, by which the mean Diameter of any Casks may be readily found:

To find the Ullage of a Cask.

Problem VI.

Let Figure 21 represent a Cask, posited with its Axis $e f$, Parallel to the Horizon, in part empty, the Surface of the Liquor cutting the Bung-Diameter in S , suppose the whole Content 97.455 Gallons, the Bung-Diameter $a b$, 32 Inches, the wet part $s b$, 24, the dry part $S a$, 8; How many Gallons are contained in the Cask? And how many will it require to fill it full?

FOR resolving this, and such like Questions, there is a Line of Segments on the Rule, with a Line of Numbers to slide against it, and upon these Lines the Proportion is:

1. As the Bung-Diameter upon the Numbers, is to 100 upon the Segments;

So is the wet or dry parts in the Numbers, to a fourth Number on the Segments.

2. As 100 upon C , is to the Cask's whole Content upon A ;

So is the fourth Number last found to the Answer.

Sect. IX. Of Cask-Gauging. 113

Example. 1. Set 32 the Bung-Diameter on the Numbers, to 100 on the Segments lying, then against 8, the dry Inches on the Numbers, is 17.4 on the Segments.

2. Set 100 upon B, to 97.455; the Cask's whole Content upon A, then against 17.4 upon B, you have 16.905 upon A, and so many Gallons it will take to fill up the Cask; this subtracted from 97.455, the whole Content, there rests 80.555, the quantity of Liquor in the Cask; which may otherwise be found thus, having set 32 the Bung-Diameter to 100 on the Segments, against 24 the wet Inches on the Numbers, you have 82.25 on the Segments.

Then,

As 100 upon B, is to 97.455 upon A;

So is 82.25 upon B, to 80.555 upon A: the same as above.

Note. The Line of Segments on this Rule is made for a *Sphaeroid*, and is therefore more exact for all Bulging Casks than the Table of Segments, which are proper only for a Cylinder.

The following Tables shew the Ullage of two several Casks in Gallons and Quarts: They were made by drawing out the Liquor by an exact Wine-Quart, and the dry Inches taken by an Instrument proper for that purpose.

0	12	100	1	8	100
0	24	100	1	16	100
0	36	100	1	24	100
0	48	100	1	32	100
0	60	100	1	40	100
0	72	100	1	48	100
0	84	100	1	56	100
0	96	100	1	64	100
0	108	100	1	72	100
0	120	100	1	80	100
0	132	100	1	88	100
0	144	100	1	96	100
0	156	100	1	104	100
0	168	100	1	112	100
0	180	100	1	120	100
0	192	100	1	128	100
0	204	100	1	136	100
0	216	100	1	144	100
0	228	100	1	152	100
0	240	100	1	160	100
0	252	100	1	168	100
0	264	100	1	176	100
0	276	100	1	184	100
0	288	100	1	192	100
0	300	100	1	200	100

A

A Table showing the Volume of a Cask in Gallons and Quarts, the Length of the Cask is 37.3 Inches, the Bung-Diameter 26.5, the Head-Diameter 25, and the Content 100 Wine-Gallons.

Dry Inches.	Gall. & Quarts.	Dry Inches.	Gall. & Quarts.	Dry Inches.	Gall. & Quarts.
0.71	1	6.19	13 2	10.98	32 2
1.10	2	6.33	14 0	11.09	33 0
1.49	3	6.48	14 2	11.20	33 2
1.62	1 0	6.62	15 0	11.31	34 0
1.82	1 1	6.76	15 2	11.42	34 2
2.00	1 2	6.90	16 0	11.53	35 0
2.16	1 3	7.04	16 2	11.64	35 2
2.30	2 0	7.18	17 0	11.75	36 0
2.44	2 1	7.32	17 2	11.86	36 2
2.57	2 2	7.46	18 0	11.97	37 0
2.69	2 3	7.60	18 2	12.08	37 2
2.80	3 0	7.73	19 0	12.19	38 0
2.91	3 1	7.87	19 2	12.30	38 2
3.01	3 2	8.00	20 0	12.41	39 0
3.11	3 3	8.14	20 2	12.52	39 2
3.20	4 0	8.26	21 0	12.61	40 0
3.30	4 1	8.39	21 2	12.74	40 2
3.39	4 2	8.52	22 0	12.85	41 0
3.48	4 3	8.65	22 2	12.96	41 2
3.57	5 0	8.77	23 0	13.07	42 0
3.65	5 1	8.90	23 2	13.18	42 2
3.73	5 2	9.02	24 0	13.28	43 0
3.81	5 3	9.14	24 2	13.39	43 2
3.89	6 0	9.26	25 0	13.49	44 0
4.05	6 2	9.38	25 2	13.60	44 2
4.21	7 0	9.50	26 0	13.70	45 0
4.37	7 2	9.62	26 2	13.81	45 2
4.53	8 0	9.73	27 0	13.92	46 0
4.69	8 2	9.85	27 2	14.03	46 2
4.84	9 0	9.96	28 0	14.12	47 0
4.99	9 2	10.08	28 2	14.23	47 2
5.15	10 0	10.19	29 0	14.33	48 0
5.30	10 2	10.31	29 2	14.44	48 2
5.45	11 0	10.42	30 0	14.54	49 0
5.60	11 2	10.54	30 2	14.65	49 2
5.75	12 0	10.65	31 0	14.75	50 0
5.90	12 2	10.76	31 2		
6.04	13 0	10.87	32 0		

A Table shewing the Ullage of a Cask to every half Gallon, the Length of the Cask is 30.8 Inches, the Bung-Diameter 25.6, the Head-Diameter 22.6, and the Content 64 Wine-Gallons.

Dry Inch.	Gall. and Quart	Dry Inch.	Gall. and Quart	Dry Inch.	Gall. and Quart
.80	2	5.76	11 2	9.60	22 2
1.25	1 7	5.94	12 0	9.77	23 0
1.60	1 2	6.12	12 2	9.94	23 2
1.91	2 0	6.30	13 0	10.11	24 0
2.17	2 2	6.48	13 2	10.28	24 2
2.40	3 0	6.66	14 0	10.45	25 0
2.63	3 2	6.84	14 2	10.62	25 2
2.85	4 0	7.01	15 0	10.79	26 0
3.07	4 2	7.19	15 2	10.96	26 2
3.28	5 0	7.36	16 0	11.13	27 0
3.49	5 2	7.54	16 2	11.30	27 2
3.69	6 0	7.71	17 0	11.47	28 0
3.89	6 2	7.89	17 2	11.64	28 2
4.08	7 0	8.06	18 0	11.81	29 0
4.27	7 2	8.24	18 2	11.98	29 2
4.46	8 0	8.41	19 0	12.14	30 0
4.65	8 2	8.58	19 2	12.31	30 2
4.84	9 0	8.75	20 0	12.47	31 0
5.03	9 2	8.92	20 2	12.64	31 2
5.21	10 0	9.09	21 0	12.80	32 0
5.40	10 2	9.26	21 2		
5.58	11 0	9.43	22 0		

The Casks by which these Tables were made, were a Brandy-Punchion, and a Brandy-Hoghead, both from Bourdeaux: The Tables will serve indifferently for all such Casks, whose Diameters, Length, and Contents are the same with these, and are so plain, that they need no Example to shew their use; for, against the dry Inches you have the Gallons and Quarts, that the Cask wants of being full, and if the Bung-Diameter be more than half dry, enter the Table with the wet Inches instead of the dry, it shews the quantity of Liquor in the Cask: These Tables may by Proportion be converted into Lines, and put upon a Rule.

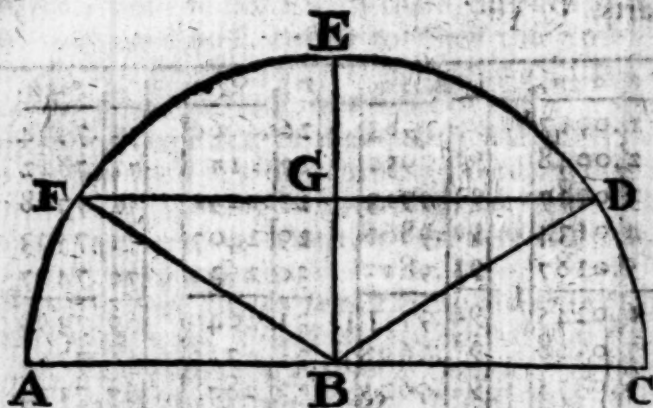
By what hath been already said, 'tis easie (I presume) to find the Ullage of a Cask whose Axis is posited Parallel to the Horizon: But lest this Tract should be thought imperfect, I shall here insert a Table of the Segments of a Circle, and also shew their Construction and Use.

A TABLE of the Segments of a Circle whose Area is Unity, the Diameter (viz. 1.128378) being divided into 100 equal parts.

v.S. Segm.	v.S. Segm.	v.S. Segm.	v.S. Segm.
1.0017	99.9983	26.2066	74.7934
2.0048	98.9952	27.2178	73.7822
3.0087	97.9913	28.2292	72.7708
4.0134	96.9866	29.2407	71.7593
5.0187	95.9813	30.2523	70.7477
6.0245	94.9755	31.2640	69.7359
7.0308	93.9692	32.2759	68.7241
8.0375	92.9625	33.2878	67.7124
9.0446	91.9554	34.2998	66.7002
10.0520	90.9480	35.3119	65.6881
11.0598	89.9402	36.3241	64.6759
12.0680	88.9320	37.3364	63.6636
13.0764	87.9236	38.3487	62.6513
14.0851	86.9149	39.3611	61.6389
15.0941	85.9059	40.3735	60.6265
16.1033	84.8967	41.3860	59.6140
17.1127	83.8873	42.3986	58.6014
18.1224	82.8776	43.4112	57.5888
19.1323	81.8677	44.4238	56.5762
20.1424	80.8576	45.4364	55.5636
21.1526	79.8474	46.4491	54.5509
22.1621	78.8366	47.4618	53.5382
23.1737	77.8263	48.4745	52.5255
24.1845	76.8155	49.4873	51.5127
25.1055	75.8044	50.5000	50.5000

The

The Construction of the Table of Segments of a Circle.



In the Figure here before us, all that part or portion comprehended within the Right Lines DB and BF, and the Circular Line FED, is called the Sector of a Circle, FGD is the Cord of the Segment, GE is the Versed Sine, and GB the Complement thereof, ABC is the Diameter of the Circle, which here we suppose to be 2.

Now, if the Diameter of a Circle be 2, the Length of the Circumference is double the Area of the Circle, for the Circumference is 6.2831852, and the Area 3.1415926; hence it evidently follows, that the Length or half the Arch of any Sector, of this Circle, is equal to the Area of that Sector, and (as shall be proved anon) the Natural Sine of the said half Arch is equal to the Area of the Triangle in that Sector.

Now, the Area of the Triangle, taken from the Area of the Sector, leaves the Area of the Segment,

Example.

Example. Let it be required to find the Area of the Segment FEDG, whose versed Sine GE is .41

1. To find the Area of the Sector.

In order to which we must first know the Quantity of the Angle at the Center, and that is thus discovered.

Seek the Complement of the versed Sine in the Table of Natural Sines, the Degrees and Decimal Parts (or Minutes) thereunto belonging are the Cosine of half the Angle at the Center.

Thus BE 1, -- GE .4 = BG .6, this found in the Table of Natural Sines, gives 36 Degrees and 87 Parts, the Angle ABE, its Complement is 53 13 the Arch FE, which doubled is 106 26 the Arch of the Sector FED, that is, the Quantity of the Angle at the Center.

Now to find the Length of any Number of Degrees, (that is, the Length of any Arch) of this Circle, Say,

As 360, the Degrees in the whole Circle, is

6.2831852, the Length of the whole Circumference,

So is 1, to 0.1745329, the Length of one Degree.

And this is a common Multiplicator for any Number of Degrees; so the Degrees in the Arch FE, viz. 53 13, being Multiplied by .01745329 gives .9272932977, the Length of the Arch FE, that is, the Area of the Sector BFED, which was required.

G

2. To find the Area of the Triangle BFD.

The Diameter being .2, I say, half the Natural Sine of the Angle at the Center, when the said Angle does not exceed 90 $^{\circ}$. when it does, then half the Natural Sine of its Complement to 180, is equal to the Area of the Triangle in the Sector.

Thus in the Sector BFED, the Angle at the Center is 106.26, its Complement to 180 is 73.74, whose Natural Sine by the Table is .96, the half of this is .48, the Area of the Triangle BFD, which was required.

The truth of which doth thus appear, BF is .1, BG is .6, ergo GF is .8, for $BF^2 + BG^2 = EG^2$ (by 47. 1 of Euclid.)

But FG .8 Multiplied by BG .6 gives .48 the Area of the Triangle BFD as before. Then

From the Area of the Sector BFED .91729329

Take the Area of the Triangle BFD. 48000000

There remains the Area of the Segment FEDG .43729329

But seeing in all Tables showing the Area's of the Segments of a Circle, it is supposed the Area of the whole Circle is Unity, the foregoing Table was made by this Proportion.

As the Area of that Circle whose Diameter is 1, viz. 3.1415926, is to the Segment of any part of that Diameter :

So is 1 (the suppose Area of another Circle) to the Segment of the like part of its Diameter. Thus, As 3.1415926, is to .43729329 the Segment last found,

Sect. IX. of the Table of Segments. 123

So is 1 to 14277, the Segment answering to the versed Sine 20, in the Table.

For as the versed Sine of the first Segment, viz. GE was .4 that is $\frac{2}{5}$ of the Radius BE;

So 20 the versed Sine of this last Segment is $\frac{2}{5}$ of the Radius of the Table.

The Use of the TABLE of Segments,

Is to find the Ullage or Quantity of Liquor remaining in a Cask, whose Axis is posited Parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask; and for this the Rule is;

To the wet or dry Inches of the Bung-Diameter add a competent number of Cyphers, then divide it by the whole Diameter, the Quotient found in the Table under Title VS. gives a Segment, which multiplied by the whole Content of the Cask, the Product shews the quantity of Liquor in the Cask, if the Dividend was the wet Inches, or the Ullage if it was the dry.

Example. Let there be a Cask in the form of a Cylinder, whose Bung-Diameter is 33 Inches, the dry part 12, and the Content of the Cask 108 Gallons: How many Gallons are wanting to fill up the Cask?

12 Divided by 33 quotes 36, which found in the Table under VS gives 3241; this Segment multiplied by 108 (the whole Content of the Cask) the Product is 35, and so many Gallons the Cask wants of being full.

Note, If a Cask be in the form of a Cylinder, or near that Figure, the Table will give the Ullage more exactly than the Line of Segments on the Rule:

G 2

But

but if the Bung-Diameter be much greater than the Head-Diameter, the Line of Segments is truer than the Table.

Problem VII. Figure 23.

There is a Cask supposed to be the middle Frustum of a Spheroid, standing upon one Head, with its Axis Perpendicular to the Horizon, the Dimensions

Are { E H Length ————— { 24
 { AB Bung-Diameter — { 20 } Inches.
 { C D Head-Diameter — { 16 }

To calculate any Diameter betwixt the Bung and Head (suppose SV) and by such Diameter to find the Quantity of Liquor in the Cask when part is empty.

1. To find the Diameter the Rule is,

From the Square of the Bung-Diameter (A B) subtract the Square of the Head-Diameter (C D) and divide the Square Root of the Remainder by half the Cask's Length (E O); then multiply this Quotient by the Number of Inches, that the Diameter sought is distant from the Bung, and call this Product your Subducend. Lastly, from the Square of the Bung-Diameter, subtract the Square of your Subducend, the Square Root of the Remainder is the Diameter sought.

Example

Example. Let it be required to find the Diameter SV, which is supposed to be 6 Inches distant from AB. the Bung-Diameter.

The Square } Bung-Diameter is ——— 400
of the } Head-Diameter is ——— 256

40 The Remainder is 21 5000 10 1000 144

Whole Square-Root is 12, this, divided by half the Cask's Length, $\frac{1}{2}$ 12, the Quotient is 1, this multiplied by 6, for the Subducend.

Square of the Bung-Diameter ——— 400
Square of the Subtend ——— 36

There remains 364

Which is the Square of the Diameter SV. its Square Root is 19.07878, the Diameter it self; but the Square of it is sufficient for our purpose.

2. To find the Quantity of Liquor in the Cask.

Having found the Square of SV, the Diameter of the Liquor's Surface, the Quantity of Liquor is easily attained :

For by the Rule laid down in the first Problem of this Section, the whole Content of the Cask will be found to be 23.532 Ale-Gallons, the half of this is 11.766, for the Content of the lower half, A n t B, and part ASVB will be found to be 6.484 Gallons.

For the Square of AB is ——— 400

The Square of SV is ——— 364

Sum is ——— 764

Half Sum is ——— 382

Half the Difference of the Squares is 18

Sum of these is ——— 1164

This multiplied by *e.g.* viz. 6, gives 6984, and this divided by 1077 quotes 6.484, the Content of the part ASVB, as above; now this added to the Content of the lower half, viz. 11.766, gives 18.25, and so many Gallons are in the Cask: If this be taken from 23.532, the whole Content, there rests 5.282, and so many Gallons are required to fill up the Cask: for this 5.282 Gallons, is the Content of the part SCDV.

If the Cask were less than half full, the Ullage might be found by the same method, and that subtracted from the whole Content leaves the Quantity of Liquor in the Cask.

SECT.

SECT. X.

Of Gauging of Vorks in open Vessels

FOR this purpose there are two Lines placed on the under side of the two Sliding Pieces of the Rule, which being so plain and obvious as not to need any Description, I shall proceed to shew their Use, which is to find how many Ale-Gallons, and Hundred parts of a Gallon, any small Tub, or such like open Vessel (from 12 to 36 Inches Diameter) will contain at one Inch deep; to perform which observe the following Directions.

 Let the Sliding Piece CD, be pin'd fast at the end towards the Right-Hand; then draw out the other Sliding Piece, 'till the two ends touch any two opposite sides of the Tub or Vessel, then look what Divisions (both upon the Line of Gallons and Line of Inches upon the Sliding Piece) are cut by the end of the Rule, for that Point doth shew the Diameter of the Tub in Inches and tenth Parts upon one Line; and upon the other, how many Gallons and hundred parts it will contain at one Inch deep.

Thus when the end of the Rule cuts 13.4 on the Line of Inches, it also cuts 0 Gallons; tenths on the Line of Gallons; in like manner, when it cuts 21 on the Inch-Line, it will at the same time cut 1.33 on the Line of Gallons; that is, 1 Gallon and .33 hundred parts; and so much will a Tub

of 21 Inches Diameter contains at one Inch deep. And thus you may draw out this Piece to 24 Inches and an half, and then the end of the Rule will cut 1.673 on the Line of Gallons.

Now if the Diameter of the Tub be more than 24.5, put this Piece at 24.5, and then draw out the other Sliding Piece at the other end, where you will find, that when the end of the Rule cuts 30 Inches, it will also cut 2.51 Gallons; and thus you will find 3 Gallons, 6 tenths and more. *Note*, In all these Cases 'tis the Area only which is sought; the Inches do also shew the Diameter, but this is to make the other more plain.

Now the Tubs or Vessels which Victuallers commonly make use of for cooling their Worts, and working their Ale, are generally Cylindrical or Conical, and in either of these the Bases are Circular or Elliptical.

First, of such as are Cylinders, or near that Form.

If the Bases of a Cylindrical Tub be circular, the Rule shews the Area or Content at one Inch deep by Inspection, (as above) this multiplied by the Depth of the Liquor gives the Content.

For instance.

Admit I come to a Tub, and (by the Rule) find the Area to be 1 Gallon 4 tenths, and the Depth of the Wort 17 Inches and an half:

Multiply

Multiply the Area 1.4
by the Depth; 7.5

The Product is 10.50

That is, 10 Gallons and an half, which is the Content sought.

But this Content may be found without the Pen; for having the Area and the Depth, you may multiply the one by the other, by the Rule: Thus,

Set 1 upon B, to 1.4 the Area upon A; then against 7.5 the Depth upon B, you have 10.5 upon A, the Content as before.

If the Vessel be Elliptical, take the cross Diameters in the middle of the Depth; as suppose the longest Diameter is 60 Inches; and the shortest 40; say,

As 359 is to one of the Diameters;

So is the other Diameter to the Area.

And by this Proportion on the Lines A and B, the Area of the Ellipsis above-mentioned will be found to be 6.68: For,

As 359 upon B, is to 40 upon A;

So is 60 upon B, to 6.68 upon A.

This is the true way of finding the Area of an Ellipsis. But in small Elliptical Tubs you may find the Area with less trouble: Thus,

Find (by drawing out the Rule) the Area of the longest Diameter, and in like manner the Area of the shortest Diameter, half the Sum of these two is the Area sought.

For Instance.

Suppose the Area of the longest Diameter be 3.62
 And the Area of the shortest ————— 2.51
 The Sum is ————— 6.13
 The half is ————— 3.065

The Area, required, which is very near the truth;
 for if you work by the former Rule, you will find
 the true Area to be 3.008.

Secondly, for Conical Vessels.

When the Diameters at Top and Bottom differ
 not above 4 Inches, you may find the Area in the
 middle of the Depth, by drawing out the Rule as
 above directed, and this Area thus found is very
 near a true Mean for the whole Tub.

Example. Let the Diameter at the Bottom of a
 Conical Tub be 24 Inches, and the Diameter at the
 Top 28 Inches, in the middle of the Depth of this
 Tub you will find (by the Rule) the Diameter 26
 Inches, and the Area 1.88 Gallons.

And for such Conical Tubs (or Tuns) whose Dia-
 meters are above 36 Inches, if they be near a Cylin-
 der, one mean Inch will be sufficient.

But for such as are much Conical and great, you
 may find the whole Content by Problem V. SECT. VII.
 And if you desire to Inch any such Tun, observe
 the Directions in Problem I. SECT. VIII.

There are yet another sort of Vessels which Visu-
 allers do frequently use to work their Ale in, and
 these are part of a Butt or Pipe set upon one Head,
 the others being cut off. The

The whole Content of these may be found by Problem VII. Section IX.

I have observed in several Places they cool their Worts in Brass-Pans, whose Diameter is about 30 Inches, and Depth 10 or 12 Inches; but these are commonly deeper in the middle by an Inch or two, than at the Side.

In such Vessels as these, if you take (with the Rule) the Area of the Liquor's Surface, and multiply it by the Depth taken within 3 or 4 Inches of the Side, the Product will be the Content very near the truth.

And for round Bowls, which are like the Segment of a Globe, if you multiply the Area of the Liquor's Surface by half the Depth, it gives the Content near enough in Practice: But in such Casts as these, Experience will be your best Tutor.

But that I may not be wanting to the meanest Capacities, I have here inserted a very useful Table, shewing the Area's of Circles, and Contents of Cylinders in Ale-Gallons.

The first Column of the Table shews the Diameters of the Circles, and the second Column shews the Areas of the Circles, and the third Column shews the Contents of the Cylinders, in Ale-Gallons, for every Inch of Depth.

The other three Columns shew the Contents of the Cylinders, in Ale-Gallons, for every Inch of Depth, when the Diameter of the Circle is 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100.

The



THE
Explanation and Use
OF THE
Table of Cylinders.

THE Table consists of two Parts: The first shews the *Area's* of *Circles*, and Contents of *Cylinders*, from 12 to 42 Inches Diameter, and from 1 to 10 Inches deep, (and by Addition, to any other Depth whatsoever.) The second Part shew only the *Area's* of *Circles* from 42 to 156 Inches Diameter.

In the first Part of the Table, the two first Pages, (and so every two next succeeding) contain 10 Columns: In the first of which you have the Diameters of Circles beginning at 12, and continued down (in the said first Column on every left-hand Page) in Inches and Tenths, to 42 Inches.

The other Nine Columns are number'd at the Top, 1, 2, 3, 4, &c. to 9. In the first of these under 1, you have (against their respective Diameters) the *Area's* of *Circles* (or the Content at 1 Inch deep) to the hundred thousandth part of an Ale-Gallon.

of the Table of Cylinders. 133

In the next Column under 2, is the Content at 2 Inches deep; under 3, the Content at 3 Inches deep; and so on to 9 Inches deep in the last Column under 9.

Thus if the Diameter of a Circle be 29.6 Inches the Area is 2.4401 Gallons, the Content at 1 Inch deep. And if the Diameter of a Cylinder be 29.6 Inches,

The Content at	2	Inches Deep is	4.88	Gallons.
	3		7.32	
	4		9.76	
	5		12.20	
	6		14.64	
	7		17.08	
	8		19.52	
	9		21.96	

Note, If the former Cylinder of 29.6 Inches Diameter,

Be the Content	10	Inches Deep, the	24.4	Gallons.
	20		48.8	
	30		73.2	
	40		97.6	
	50		122.0	
	60		146.4	
	70		170.8	
	80		195.2	
	90		219.6	

For the Content of any Cylinder at 2 Inches, and 20 Inches deep, is express'd by the same Figures; but as 20 is ten times 2, the Area at 2 Inches deep must be made ten times greater, which is done by only putting the Prick, or Point one

134 *The Explanation and Use, &c.*

one place more towards the Right-hand. From hence the Use of this Table is obvious, as may further appear by the following Example.

There is a Tun in the form of a Cylinder, the Diameter is 29.6 and the Depth 27.8 Inches, what is the Content in Gallons?

At 29.6 Inches Diameter,

20 Inches deep is	— — — — —	48.80
7 Inches deep is	— — — — —	17.08
.8 of an Inch deep is	— — — — —	1.95
27.8 Content sought is	— — — — —	67.83

If the mean Diameter of your Tun be above 52 Inches, find it in the first Column of the second part of the Table; for against the whole Inches of your Diameter in the side of the Table, and under the tenths (if any be) you have the Area or Content at one Inch deep; this multiply'd by the whole Depth, gives the Content of the Tun; but this is so plain it needs no Example.

And for Casks, find the mean Diameter by the Direction given in Page 121, and then by the Length and mean Diameter the Content is found as above.

So if the Length of a Cask be 30 Inches, and the mean Diameter 23 Inches, the Content will be found to be 44.2 Gallons; for against 23, the mean Diameter, and under 3 is 44.2. Now if 3 be 30, 44.2 will be 44.2, the Content sought.

A
TABLE
OF
CYLINDERS,

From 12, to 42 Inches Diameter,
and from 1, to 9 Inches Deep ;

Very useful in Gauging of

Small Tuns and Casks ;

S H E W I N G

By Inspection, the *Content* of such
Vessels to the Thousandth
Part of a Gallon, Ale-Mea-
sure,

136 *A Table of the Area's of Circles.*

Diam.in Inches.	D E P T H.			
	1	2	3	4
12.0	0.4011	0.80	1.20	1.60
.1	0.4078	0.81	1.22	1.63
.2	0.4146	0.83	1.24	1.65
.3	0.4214	0.84	1.26	1.69
.4	0.4282	0.86	1.28	1.71
.5	0.4352	0.87	1.30	1.74
.6	0.4422	0.88	1.33	1.77
.7	0.4492	0.90	1.35	1.80
.8	0.4563	0.91	1.37	1.83
.9	0.4635	0.93	1.39	1.85
13.0	0.4707	0.94	1.41	1.88
.1	0.4780	0.96	1.43	1.91
.2	0.4853	0.97	1.46	1.94
.3	0.4927	0.98	1.48	1.97
.4	0.5001	1.00	1.50	2.00
.5	0.5076	1.01	1.52	2.03
.6	0.5151	1.03	1.55	2.06
.7	0.5227	1.04	1.57	2.09
.8	0.5304	1.06	1.59	2.12
.9	0.5381	1.08	1.61	2.15
14.0	0.5459	1.09	1.64	2.18
.1	0.5537	1.11	1.66	2.21
.2	0.5616	1.12	1.68	2.24
.3	0.5695	1.14	1.71	2.28
.4	0.5775	1.15	1.73	2.31
.5	0.5856	1.17	1.76	2.34
.6	0.5937	1.19	1.78	2.37
.7	0.6018	1.20	1.80	2.40
.8	0.6100	1.22	1.83	2.44
.9	0.6183	1.24	1.85	2.47

DEPTH				
5	6	7	8	9
2.01	2.41	2.81	3.21	3.61
2.04	2.44	2.85	3.26	3.67
2.07	2.49	2.90	3.31	3.73
2.11	2.53	2.95	3.37	3.79
2.14	2.57	3.00	3.43	3.86
2.18	2.61	3.05	3.48	3.92
2.21	2.65	3.10	3.54	3.98
2.24	2.69	3.14	3.59	4.04
2.28	2.73	3.19	3.65	4.10
2.32	2.78	3.24	3.71	4.17
2.35	2.82	3.30	3.77	4.24
2.39	2.87	3.35	3.82	4.30
2.43	2.91	3.39	3.88	4.36
2.46	2.95	3.44	3.94	4.43
2.50	3.00	3.50	4.00	4.50
2.53	3.04	3.55	4.06	4.56
2.58	3.09	3.61	4.12	4.63
2.61	3.13	3.65	4.18	4.70
2.65	3.18	3.71	4.24	4.77
2.69	3.23	3.77	4.30	4.84
2.73	3.27	3.82	4.36	4.91
2.76	3.32	3.87	4.42	4.98
2.81	3.37	3.93	4.49	5.05
2.84	3.41	3.98	4.55	5.12
2.88	3.46	4.04	4.62	5.19
2.93	3.51	4.10	4.68	5.26
2.96	3.56	4.15	4.74	5.34
3.01	3.61	4.21	4.81	5.41
3.05	3.66	4.27	4.88	5.49
3.09	3.71	4.33	4.94	5.56

338 *A Table of the Area's of Circles*

Diam.in Inches.	D E P T H.			
	1	2	3	4
15.0	0.6266	1.26	1.88	2.50
.1	0.6350	1.27	1.91	2.53
.2	0.6435	1.29	1.93	2.57
.3	0.6520	1.30	1.96	2.61
.4	0.6604	1.32	1.98	2.64
.5	0.6691	1.34	2.01	2.68
.6	0.6778	1.35	2.03	2.71
.7	0.6865	1.37	2.06	2.74
.8	0.6953	1.39	2.09	2.78
.9	0.7041	1.41	2.11	2.82
16.0	0.7130	1.43	2.14	2.85
.1	0.7219	1.44	2.16	2.89
.2	0.7309	1.46	2.19	2.92
.3	0.7400	1.48	2.22	2.96
.4	0.7491	1.50	2.25	3.00
.5	0.7582	1.52	2.27	3.03
.6	0.7654	1.53	2.30	3.07
.7	0.7767	1.55	2.33	3.10
.8	0.7861	1.57	2.36	3.14
.9	0.7954	1.59	2.39	3.18
17.0	0.8049	1.61	2.41	3.22
.1	0.8144	1.63	2.44	3.26
.2	0.8239	1.64	2.47	3.29
.3	0.8335	1.67	2.50	3.33
.4	0.8432	1.69	2.53	3.37
.5	0.8529	1.70	2.56	3.41
.6	0.8627	1.72	2.58	3.45
.7	0.8725	1.74	2.62	3.49
.8	0.8824	1.76	2.65	3.53
.9	0.8924	1.78	2.68	3.57

DEPTH				
5	6	7	8	9
3.13	3.76	4.38	5.01	5.63
3.18	3.81	4.45	5.08	5.72
3.22	3.86	4.50	5.15	5.79
3.26	3.91	4.56	5.22	5.87
3.30	3.96	4.62	5.28	5.94
3.35	4.01	4.68	5.35	6.02
3.39	4.06	4.74	5.42	6.09
3.43	4.12	4.80	5.49	6.18
3.48	4.17	4.87	5.56	6.26
3.52	4.22	4.93	5.63	6.34
3.57	4.28	4.99	5.70	6.42
3.61	4.33	5.05	5.78	6.50
3.65	4.39	5.12	5.85	6.58
3.70	4.44	5.18	5.92	6.66
3.75	4.49	5.24	5.99	6.74
3.79	4.53	5.31	6.06	6.82
3.84	4.60	5.37	6.14	6.90
3.88	4.66	5.43	6.21	6.99
3.93	4.71	5.50	6.28	7.07
3.98	4.77	5.57	6.36	7.16
4.02	4.83	5.63	6.44	7.24
4.07	4.88	5.70	6.51	7.33
4.12	4.94	5.77	6.59	7.42
4.17	5.00	5.83	6.66	7.50
4.22	5.06	5.90	6.74	7.59
4.26	5.11	5.97	6.82	7.68
4.31	5.17	6.03	6.90	7.76
4.36	5.23	6.10	6.98	7.85
4.41	5.29	6.17	7.06	7.94
4.46	5.35	6.24	7.14	8.03

140 *A Table of the Area's of Circles*

Diam.in Inches.	DEPTH.			
	1	2	3	4
18.0	0.9024	1.80	2.71	3.61
.1	0.9124	1.82	2.74	3.63
.2	0.9225	1.84	2.77	3.69
.3	0.9327	1.87	2.80	3.73
.4	0.9429	1.88	2.83	3.77
.5	0.9532	1.91	2.86	3.81
.6	0.9635	1.93	2.89	3.85
.7	0.9739	1.95	2.92	3.90
.8	0.9843	1.97	2.95	3.94
.9	0.9948	1.99	2.98	3.98
19.0	1.0054	2.01	3.02	4.02
.1	1.0160	2.03	3.05	4.06
.2	1.0267	2.05	3.08	4.11
.3	1.0374	2.07	3.11	4.15
.4	1.0482	2.10	3.14	4.19
.5	1.0590	2.12	3.18	4.24
.6	1.0699	2.14	3.21	4.28
.7	1.0808	2.16	3.24	4.32
.8	1.0918	2.18	3.27	4.37
.9	1.1020	2.21	3.31	4.41
20.0	1.1140	2.23	3.34	4.46
.1	1.1252	2.25	3.38	4.50
.2	1.1364	2.27	3.41	4.55
.3	1.1477	2.29	3.44	4.59
.4	1.1590	2.32	3.48	4.64
.5	1.1704	2.34	3.51	4.68
.6	1.1819	2.36	3.54	4.73
.7	1.1934	2.39	3.58	4.77
.8	1.2049	2.41	3.61	4.82
.9	1.2165	2.43	3.65	4.86

and Contents of Cylinders.

141

DEPTH.				
1.54	8.6	8.47	8.22	9.12
4.54	5.41	6.31	7.22	8.12
4.56	5.47	6.38	7.30	8.21
4.61	5.53	6.45	7.38	8.30
4.66	5.59	6.53	7.46	8.39
4.71	5.66	6.60	7.54	8.49
4.77	5.72	6.67	7.63	8.58
4.82	5.78	6.74	7.71	8.67
4.87	5.85	6.82	7.79	8.77
4.92	5.90	6.89	7.87	8.86
4.97	5.97	6.96	7.95	8.95
5.03	6.03	7.04	8.04	9.05
5.08	6.10	7.11	8.13	9.14
5.13	6.16	7.18	8.21	9.23
5.19	6.22	7.26	8.30	9.34
5.24	6.29	7.34	8.38	9.43
5.30	6.35	7.41	8.47	9.53
5.35	6.42	7.49	8.56	9.63
5.40	6.49	7.57	8.65	9.73
5.46	6.55	7.64	8.74	9.83
5.51	6.62	7.72	8.82	9.93
5.57	6.68	7.80	8.91	10.03
5.63	6.75	7.88	9.00	10.13
5.68	6.82	7.95	9.09	10.23
5.74	6.88	8.03	9.18	10.33
5.80	6.95	8.11	9.27	10.43
5.85	7.02	8.19	9.36	10.53
5.91	7.09	8.27	9.46	10.64
5.97	7.16	8.35	9.54	10.74
6.02	7.23	8.43	9.64	10.84
6.08	7.30	8.51	9.73	10.94

142 *A Table of the Areas of Circles*

Diam.in Inches.	D E P T H.			
	1	2	3	4
21.0	1.2282	2.46	3.68	4.91
.1	1.2400	2.48	3.72	4.96
.2	1.2517	2.50	3.75	5.01
.3	1.2635	2.53	3.79	5.05
.4	1.2754	2.55	3.83	5.10
.5	1.2874	2.57	3.86	5.15
.6	1.2994	2.60	3.90	5.20
.7	1.3114	2.62	3.93	5.25
.8	1.3236	2.65	3.97	5.29
.9	1.3357	2.67	4.01	5.34
22.0	1.3480	2.70	4.04	5.39
.1	1.3602	2.72	4.08	5.44
.2	1.3726	2.75	4.12	5.49
.3	1.3850	2.77	4.16	5.54
.4	1.3974	2.79	4.19	5.59
.5	1.4099	2.82	4.23	5.64
.6	1.4225	2.84	4.27	5.69
.7	1.4351	2.87	4.31	5.74
.8	1.4478	2.90	4.34	5.79
.9	1.4605	2.92	4.38	5.84
23.0	1.4733	2.99	4.42	5.89
.1	1.4861	2.97	4.46	5.94
.2	1.4990	3.00	4.50	6.01
.3	1.5120	3.02	4.54	6.05
.4	1.5250	3.05	4.58	6.10
.5	1.5380	3.08	4.62	6.15
.6	1.5511	3.10	4.65	6.20
.7	1.5643	3.13	4.69	6.26
.8	1.5775	3.15	4.73	6.31
.9	1.5908	3.18	4.77	6.36

DEPTH.				
5	6	7	8	9
6.14	7.37	8.60	9.82	11.05
6.20	7.44	8.68	9.92	11.16
6.26	7.51	8.76	10.01	11.26
6.32	7.58	8.84	10.11	11.37
6.38	7.65	8.93	10.20	11.48
6.44	7.72	9.01	10.30	11.59
6.50	7.80	9.09	10.39	11.69
6.56	7.87	9.18	10.49	11.80
6.62	7.94	9.26	10.59	11.91
6.68	8.01	9.35	10.61	12.02
6.74	8.09	9.44	10.78	12.13
6.80	8.16	9.52	10.88	12.24
6.86	8.23	9.61	10.98	12.35
6.93	8.31	9.70	11.08	12.47
6.99	8.38	9.78	11.18	12.58
7.05	8.46	9.87	11.28	12.69
7.11	8.53	9.96	11.38	12.80
7.18	8.61	10.05	11.48	12.92
7.24	8.68	10.13	11.58	13.02
7.30	8.76	10.22	11.68	13.14
7.37	8.84	10.31	11.78	13.26
7.43	8.92	10.40	11.89	13.37
7.50	8.99	10.49	11.99	13.49
7.56	9.07	10.58	12.10	13.61
7.63	9.15	10.68	12.20	13.73
7.69	9.23	10.77	12.30	13.84
7.76	9.31	10.86	12.41	12.96
7.82	9.39	10.95	12.51	14.08
7.89	8.46	11.04	12.62	14.23
7.95	9.54	11.13	12.72	14.31

144 *A Table of the Area's of Circles*

Diam.in Inches.	D E P T H.			
	1	2	3	4
24.0	1.6042	3.21	4.81	6.42
.1	1.6176	3.23	4.85	6.47
.2	1.6310	3.26	4.89	6.52
.3	1.6445	3.29	4.93	6.58
.4	1.6581	3.32	4.97	6.63
.5	1.6717	3.34	5.01	6.69
.6	1.6854	3.37	5.06	6.74
.7	1.6991	3.40	5.10	6.80
.8	1.7129	3.43	5.14	6.85
.9	1.7267	3.45	5.18	6.91
25.0	1.7406	3.48	5.22	6.96
.1	1.7546	3.51	5.26	7.02
.2	1.7686	3.54	5.31	7.07
.3	1.7827	3.57	5.35	7.13
.4	1.7968	3.59	5.39	7.19
.5	1.8110	3.62	5.43	7.24
.6	1.8252	3.65	5.48	7.30
.7	1.8395	3.68	5.52	7.36
.8	1.8538	3.71	5.56	7.42
.9	1.8682	3.74	5.60	7.47
26.0	1.8827	3.77	5.65	7.53
.1	1.8972	3.79	5.69	7.59
.2	1.9117	3.82	5.73	7.65
.3	1.9264	3.85	5.78	7.71
.4	1.9411	3.88	5.82	7.76
.5	1.9558	3.91	5.87	7.82
.6	1.9706	3.94	5.91	7.88
.7	1.9854	3.97	5.96	7.94
.8	2.0003	4.00	6.00	8.00
.9	2.0153	4.03	6.05	8.06

DEPTH.				
5	6	7	8	9
8.02	9.62	11.23	12.83	14.44
8.09	9.70	11.32	12.93	14.55
8.16	9.79	11.42	13.05	14.68
8.22	9.86	11.51	13.15	14.80
8.29	9.95	11.61	13.26	14.92
8.36	10.03	11.70	13.37	15.04
8.43	10.11	11.80	13.48	15.17
8.50	10.19	11.89	13.59	15.29
8.56	10.28	11.99	13.70	15.42
8.63	10.36	12.08	13.81	15.54
8.70	10.44	12.18	13.92	15.67
8.77	10.53	12.28	14.03	15.79
8.84	10.61	12.38	14.15	15.91
8.91	10.70	12.47	14.26	16.04
8.98	10.78	12.57	14.37	16.17
9.06	10.87	12.68	14.49	16.30
9.13	10.95	12.78	14.60	16.43
9.20	11.04	12.88	14.71	16.55
9.27	11.12	12.98	14.83	16.64
9.34	11.21	13.08	14.94	16.81
9.41	11.29	13.18	15.06	16.94
9.49	11.38	13.28	15.18	17.07
9.56	11.47	13.38	15.29	17.20
9.63	11.56	13.48	15.41	17.34
9.70	11.65	13.59	15.53	17.47
9.78	11.73	13.69	15.65	17.60
9.85	11.82	13.79	15.76	17.74
9.93	11.91	13.90	15.88	17.87
10.00	12.00	14.00	16.00	18.00
10.08	12.09	14.11	16.12	18.14

H

146 *A Table of the Area's of Circles*

Diam.in	D E P T H.			
Inches.	1	2	3	4
27.0	2.0303	4.06	6.09	8.12
.1	2.0454	4.09	6.14	8.18
.2	2.0605	4.12	6.18	8.24
.3	2.0757	4.15	6.23	8.30
.4	2.0909	4.18	6.27	8.36
.5	2.1062	4.21	6.32	8.42
.6	2.1215	4.24	6.36	8.48
.7	2.1369	4.27	6.41	8.55
.8	2.1524	4.30	6.46	8.61
.9	2.1679	4.33	6.50	8.67
28.0	2.1835	4.37	6.55	8.73
.1	2.1991	4.40	6.60	8.80
.2	2.2148	4.43	6.64	8.86
.3	2.2305	4.46	6.69	8.92
.4	2.2463	4.49	6.74	8.98
.5	2.2621	4.52	6.79	9.05
.6	2.2780	4.56	6.83	9.11
.7	2.2940	4.59	6.88	9.18
.8	2.3100	4.62	6.93	9.24
.9	2.3261	4.65	6.98	9.30
29.0	2.3422	4.68	7.03	9.37
.1	2.3584	4.72	7.07	9.43
.2	2.3746	4.75	7.12	9.50
.3	2.3910	4.78	7.17	9.56
.4	2.4073	4.81	7.22	9.63
.5	2.4237	4.85	7.27	9.69
.6	2.4401	4.88	7.32	9.76
.7	2.4567	4.91	7.37	9.83
.8	2.4732	4.94	7.42	9.89
.9	2.4899	4.98	7.47	9.96

DEPTH.

5	6	7	8	9
10.15	12.18	14.21	16.24	18.27
10.23	12.27	14.32	16.36	18.41
10.30	12.36	14.42	16.48	18.54
10.38	12.45	14.53	16.61	18.68
10.45	12.55	14.64	16.73	18.82
10.53	12.64	14.74	16.85	18.95
10.61	12.73	14.85	16.97	19.09
10.68	12.82	14.96	17.10	19.23
12.76	12.91	15.06	17.22	19.37
10.84	13.01	15.18	17.34	19.51
10.92	13.10	15.28	17.46	19.65
11.00	13.19	15.39	17.59	19.79
11.08	13.29	15.50	17.71	19.93
11.15	13.38	15.61	17.84	20.07
11.23	13.48	15.72	17.97	20.22
11.31	13.57	15.83	18.10	20.36
11.39	13.67	15.95	18.22	20.50
11.47	13.76	16.06	18.35	20.65
11.55	23.86	16.17	18.48	20.79
11.63	13.96	16.28	18.61	20.93
11.71	14.05	16.39	18.74	21.08
11.79	14.15	16.51	18.86	21.22
11.87	14.25	16.62	18.99	21.37
11.96	14.35	16.74	19.13	21.52
12.04	14.44	16.85	19.26	21.66
12.12	14.54	16.96	19.39	21.81
12.20	14.64	17.08	19.52	21.96
12.28	14.74	17.19	19.65	22.10
12.37	14.84	17.31	19.78	22.26
12.45	14.94	17.43	19.92	22.41

Diam.in Inches.	D E P T H.			
	1	2	3	4
30.0	2.5066	5.01	7.52	10.03 ¹
.1	2.5233	5.05	7.57	10.09 ¹
.2	2.5401	5.08	7.62	10.16 ¹
.3	2.5570	5.11	7.67	10.23 ¹
.4	2.5739	5.15	7.72	10.29 ¹
.5	2.5908	5.18	7.77	10.36 ¹
.6	2.6078	5.21	7.82	10.43 ¹
.7	2.6249	5.25	7.87	10.50 ¹
.8	2.6420	5.28	7.93	10.57 ¹
.9	2.6592	5.32	7.98	10.64 ¹
31.0	2.6764	5.35	8.03	10.71 ¹
.1	2.6938	5.39	8.08	10.78 ¹
.2	2.7111	5.42	8.13	10.84 ¹
.3	2.7285	5.46	8.19	10.91 ¹
.4	2.7460	5.49	8.24	10.98 ¹
.5	2.7635	5.53	8.29	11.05 ¹
.6	2.7811	5.56	8.34	11.12 ¹
.7	2.7987	5.60	8.40	11.19 ¹
.8	2.8163	5.63	8.45	11.27 ¹
.9	2.8341	5.67	8.50	11.34 ¹
32.0	2.8519	5.70	8.56	11.41 ¹
.1	2.8698	5.74	8.61	11.48 ¹
.2	2.8877	5.78	8.66	11.55 ¹
.3	2.9057	5.81	8.72	11.62 ¹
.4	2.9237	5.85	8.77	11.69 ¹
.5	2.9418	5.88	8.83	11.77 ¹
.6	2.9599	5.92	8.88	11.84 ¹
.7	2.9781	5.96	8.93	11.91 ¹
.8	2.9963	5.99	8.99	11.98 ¹
.9	3.0148	6.03	9.04	12.06 ¹

DEPTH.

.5	.6	.7	.8	.9
12.53	15.04	17.54	20.05	22.56
12.62	15.14	17.66	20.18	22.71
12.70	15.24	17.78	20.32	22.86
12.79	15.34	17.90	20.46	23.01
12.87	15.44	18.02	20.59	23.17
12.95	15.55	18.14	20.73	23.32
13.04	15.65	18.25	20.86	23.47
13.12	15.75	18.37	21.00	23.62
13.21	15.85	18.49	21.14	23.78
13.30	15.95	18.61	21.27	23.93
13.38	16.06	18.73	21.41	24.09
13.47	16.16	18.86	21.55	24.25
13.56	16.27	18.98	21.69	24.40
13.64	16.37	19.10	21.83	24.56
13.73	16.48	19.22	21.97	24.71
13.82	16.58	19.34	22.10	24.87
13.91	16.69	19.47	22.25	25.03
13.99	16.79	19.59	22.38	25.18
14.08	16.90	19.71	22.53	25.35
14.17	17.00	19.84	22.67	25.51
14.26	17.11	19.96	22.82	25.67
14.35	17.22	20.09	22.96	25.83
14.44	17.32	20.21	23.10	25.98
14.53	17.43	20.34	23.24	26.15
14.62	17.54	20.46	23.39	26.31
14.71	17.65	20.59	23.53	26.48
14.80	17.76	20.72	23.68	26.64
14.89	17.87	20.85	23.82	26.80
14.98	17.98	20.97	23.97	26.97
15.07	18.09	21.10	24.11	27.13

150 *A Table of the Area's of Circles*

Diam.in Inches.	D E P T H.			
	1	2	3	4
33.0	3.0330	6.07	9.10	12.13
.1	3.0514	6.10	9.13	12.21
.2	3.0698	6.14	9.21	12.28
.3	3.0884	6.13	9.26	12.35
.4	3.1069	6.21	9.32	12.43
.5	3.1256	6.25	9.38	12.50
.6	3.1443	6.29	9.43	12.58
.7	3.1630	6.33	9.49	12.65
.8	3.1818	6.36	9.55	12.73
.9	3.2007	6.40	9.60	12.80
34.0	3.2196	6.44	9.66	12.88
.1	3.2385	6.48	9.71	12.95
.2	3.2576	6.51	9.77	13.03
.3	3.2766	6.55	9.83	13.11
.4	3.2958	6.59	9.89	13.18
.5	3.3150	6.63	9.95	13.26
.6	3.3342	6.67	10.00	13.34
.7	3.3535	6.71	10.06	13.41
.8	3.3728	6.75	10.12	13.49
.9	3.3923	6.78	10.18	13.57
35.0	3.4117	6.82	10.24	13.65
.1	3.4312	6.86	10.29	13.73
.2	3.4508	6.90	10.35	13.80
.3	3.4705	6.94	10.41	13.88
.4	3.4902	6.98	10.47	13.96
.5	3.5099	7.02	10.53	14.04
.6	3.5297	7.06	10.59	14.12
.7	3.5496	7.10	10.65	14.20
.8	3.5605	7.14	10.71	14.28
.9	3.5894	7.18	10.77	14.36

DEPTH.

5	6	7	8	8
15.17	18.20	21.13	24.26	27.30
15.26	18.31	21.36	24.41	27.46
15.35	18.42	21.49	24.56	27.63
15.44	18.53	21.62	24.71	27.79
15.53	16.64	21.75	24.85	27.96
15.63	18.75	21.88	25.00	28.13
15.72	18.86	22.01	25.15	28.30
15.82	18.98	22.14	25.30	28.47
15.91	19.09	22.27	25.45	28.63
16.00	19.20	22.40	25.60	28.80
16.10	19.32	22.54	25.75	28.97
16.19	19.43	22.67	25.90	29.14
16.29	19.54	22.80	26.06	29.32
16.38	19.66	22.93	26.21	29.49
16.48	19.77	23.07	26.36	29.66
16.58	19.89	23.21	26.52	29.84
16.67	20.00	23.34	26.67	30.01
16.77	20.12	23.47	26.82	30.18
16.86	20.23	23.61	26.98	30.35
16.96	20.35	23.75	27.14	30.53
17.06	20.47	23.88	27.29	30.70
17.16	20.59	24.01	27.45	30.88
17.25	20.70	24.16	27.61	31.06
17.35	20.82	24.29	27.76	31.24
17.45	20.94	24.43	27.92	31.41
17.55	21.06	24.57	28.08	31.59
17.65	21.18	24.71	28.24	31.76
17.75	21.30	24.85	28.39	31.94
17.85	21.42	24.99	28.55	32.13
17.95	21.53	25.12	28.71	32.30

152 *A Table of the Area's of Circles*

Diam.in	D E P T H.			
Inches.	1	2	3	4
36.0	3.6065	7.22	10.83	14.44
.1	3.6295	7.26	10.89	14.52
.2	3.6497	7.30	10.95	14.60
.3	3.6699	7.34	11.01	14.68
.4	3.6901	7.38	11.07	14.76
.5	3.7104	7.42	11.13	14.84
.6	3.7308	7.46	11.19	14.92
.7	3.7512	7.50	11.25	15.00
.8	3.7717	7.54	11.32	15.09
.9	3.7922	7.58	11.38	15.17
37.0	3.8128	7.63	11.44	15.25
.1	3.8334	7.67	11.50	15.33
.2	3.8541	7.71	11.56	15.42
.3	3.8749	7.75	11.62	15.50
.4	3.8957	7.79	11.69	15.58
.5	3.9165	7.83	11.75	15.67
.6	3.9374	7.87	11.81	15.75
.7	3.9584	7.92	11.88	15.83
.8	3.9794	7.96	11.94	15.92
.9	4.0005	8.00	12.00	16.00
38.0	4.0217	8.04	12.06	16.09
.1	4.0428	8.09	12.13	16.17
.2	4.0641	8.13	12.17	16.24
.3	4.0854	8.17	12.26	16.34
.4	4.1068	8.21	12.32	16.43
.5	4.1282	8.26	12.38	16.51
.6	4.1496	8.30	12.45	16.60
.7	4.1712	8.34	12.51	16.68
.8	4.1927	8.39	12.58	16.77
.9	4.2144	8.43	12.64	16.86

DEPTH.

5	6	7	8	
18.05	21.66	25.27	29.87	32.48
18.15	21.78	25.41	29.03	32.66
18.25	21.90	25.55	29.19	32.84
18.35	22.02	25.69	29.36	33.03
18.45	22.14	25.83	29.52	33.21
18.55	22.26	25.97	29.68	33.39
18.65	22.39	26.11	29.85	33.58
18.76	22.51	26.26	30.01	33.76
18.86	22.63	26.40	30.17	33.94
18.96	22.75	26.55	30.34	34.13
19.07	22.88	26.69	30.51	34.32
19.17	23.00	26.83	30.67	34.50
19.27	23.12	26.98	30.83	34.69
19.37	23.25	27.12	31.00	34.87
19.48	23.37	27.27	31.17	35.06
19.58	23.50	27.41	31.33	35.25
19.69	23.62	27.56	31.50	35.43
19.79	23.75	27.71	31.66	35.62
19.90	23.87	27.86	31.83	35.81
20.00	24.00	28.00	32.00	36.00
20.11	24.13	28.15	32.17	36.19
20.21	24.26	28.30	32.34	36.39
20.30	24.38	28.44	32.49	36.56
20.43	24.51	28.60	32.68	36.77
20.54	24.64	28.75	32.85	36.96
20.64	24.77	28.90	33.02	37.15
20.75	24.90	29.05	33.19	37.34
20.86	25.03	29.20	33.37	37.54
20.96	25.15	29.35	33.54	37.73
21.07	25.29	29.50	33.71	37.92

Diam.in	D E P T H.			
Inches.	1	2	3	4
39.0	4.2361	8.47	12.71	16.94
.1	4.2578	8.52	12.77	17.03
.2	4.2796	8.56	12.84	17.12
.3	4.3015	8.60	12.90	17.20
.4	4.3234	8.65	12.97	17.29
.5	4.3454	8.69	13.04	17.38
.6	4.3674	8.73	13.10	17.47
.7	4.3895	8.78	13.17	17.56
.8	4.4117	8.82	13.24	17.65
.9	4.4338	8.87	13.30	17.74
40.0	4.4562	8.91	13.37	17.82
.1	4.4785	8.96	13.43	17.91
.2	4.5008	9.00	13.50	18.00
.3	4.5233	9.05	13.57	18.09
.4	4.5457	9.09	13.64	18.18
.5	4.5683	9.14	13.71	18.27
.6	4.5908	9.18	13.77	18.36
.7	4.6135	9.23	13.84	18.45
.8	4.6362	9.27	13.91	18.54
.9	4.6589	9.32	13.98	18.64
41.0	4.6818	9.36	14.05	18.73
.1	4.7046	9.41	14.11	18.82
.2	4.7275	9.45	14.18	18.91
.3	4.7505	9.50	14.25	19.00
.4	4.7735	9.55	14.32	19.09
.5	4.7966	9.59	14.39	19.19
.6	4.8198	9.64	14.46	19.28
.7	4.8430	9.69	14.53	19.37
.8	4.8662	9.73	14.60	19.46
.9	4.8895	9.78	14.67	19.56

and Contents of Cylinders.
D E P T H.

155

.5	.6	.7	.8	.9
21.18	25.42	29.65	33.89	38.12
21.29	25.55	29.81	34.06	38.32
21.40	25.68	29.95	34.23	38.51
21.51	25.81	30.11	34.41	38.71
21.62	25.94	30.26	34.58	38.91
21.73	26.07	30.42	34.76	39.11
21.84	26.20	30.57	34.94	39.30
21.95	26.33	30.72	35.11	39.50
22.06	26.47	30.89	35.30	39.71
22.17	26.60	31.04	35.47	39.91
22.28	26.74	31.19	35.65	40.11
22.39	26.87	31.35	35.83	40.30
22.50	27.00	31.51	36.01	40.51
22.62	27.14	31.66	36.18	40.71
22.73	27.28	31.82	36.37	40.93
22.84	27.41	31.98	36.54	41.11
22.95	27.55	32.14	36.73	41.32
23.07	27.68	32.29	36.91	41.52
23.18	27.82	32.45	37.09	41.72
23.30	27.95	32.61	37.27	41.93
23.41	28.09	32.77	37.46	42.14
23.52	28.22	32.93	37.63	42.34
23.64	28.36	33.09	37.82	42.54
23.75	28.50	33.25	38.00	42.75
23.87	28.64	33.41	38.19	42.96
23.98	28.78	33.57	38.37	43.17
24.10	28.92	33.74	38.56	43.38
24.22	29.06	33.90	38.74	43.59
24.33	29.19	34.06	38.93	43.80
24.45	27.34	34.23	39.11	44.00

[illegible]

**T H E
T A B L E**

Continued,

**From 42, to 156 Inches
D I A M E T E R,**

Shewing the

A R E A ' S

O F

C I R C L E S,

T O T H E

**Ten Thousandth part of a
Gallon, Ale-Measure.**

Dia.	.0	.1	.2	.3	.4
42	4.9129	4.936	4.9598	4.9834	5.0069
43	5.1496	5.1736	5.1977	5.2218	5.2459
44	5.3920	5.4165	5.4411	5.4657	5.4904
45	5.6398	5.6649	5.6901	5.7153	5.7405
46	5.8933	5.9189	5.9446	5.9704	5.9962
47	6.1523	6.1775	6.2048	6.2311	6.2575
48	6.4169	6.4436	6.4705	6.4973	6.5243
49	6.6870	6.7143	6.7417	6.7692	6.7966
50	6.9628	6.9906	7.0186	7.0466	7.0746
51	7.2440	7.2725	7.3010	7.3295	7.3581
52	7.5309	7.5599	7.5890	7.6181	7.6472
53	7.8233	7.8529	7.8825	7.9124	7.9419
54	8.1214	8.1515	8.1816	8.2118	8.2421
55	8.4249	8.4556	8.4863	8.5171	8.5479
56	8.7341	8.7653	8.7966	8.8279	8.8593
57	9.0489	9.0806	9.1124	9.1443	9.1762
58	9.3691	9.4014	9.4338	9.4662	9.4988
59	9.6949	9.7278	9.7608	9.7938	9.8268
60	10.0264	10.0598	10.0933	10.1269	10.1605
61	10.3634	10.3974	10.4314	10.4655	10.4997
62	10.7059	10.7405	10.7751	10.8098	10.8445
63	11.0541	11.0892	11.1254	11.1596	11.1949
64	11.4078	11.4434	11.4790	11.5150	11.5508
65	11.7670	11.8033	11.8396	11.8759	11.9123
66	12.1319	12.1687	12.2055	12.2424	12.2794
67	12.5023	12.5397	12.5771	12.6145	12.6520
68	12.8773	12.9162	12.9546	12.9922	13.0303
69	13.2599	13.2982	13.3368	13.3754	13.4140
70	13.6470	13.6860	13.7251	13.7642	13.8034
71	14.0397	14.0791	14.1189	14.1586	14.1983

Gallons Ale-Measure

5	6	7	8	9
5.0306	5.0543	5.0780	5.1019	5.1257
5.2701	5.2944	5.3187	5.3430	5.3976
5.5152	5.5400	5.5649	5.5898	5.6148
5.7659	5.7912	5.8167	5.8421	5.8677
6.0221	6.0480	6.0740	6.1000	6.1261
6.2839	6.3104	6.3369	6.3635	6.3902
6.5513	6.5783	6.6054	6.6325	6.6600
6.8242	6.8518	6.8794	6.9072	6.9349
7.1027	7.1309	7.1521	7.1873	7.2157
7.3868	7.4155	7.4443	7.4731	7.5020
7.6764	7.7057	7.7350	7.7644	7.7939
7.9717	8.0015	8.0314	8.0613	8.0913
8.2724	8.3028	8.3333	8.3638	8.3943
8.5788	8.6097	8.6407	8.6708	8.7019
8.8907	8.9222	8.9538	8.9854	8.9171
9.2082	9.2403	9.2714	9.3046	9.3368
9.5313	9.5639	9.5966	9.6293	9.6621
9.8600	9.8931	9.9263	9.9596	9.9930
10.1942	10.2279	10.2617	10.2955	10.3294
10.5339	10.5682	10.6026	10.6370	10.6714
10.8793	10.9141	10.9490	10.9840	11.0190
11.2302	11.2656	11.3011	11.3366	11.3711
11.5867	11.6227	11.6587	11.6947	11.7309
11.9488	11.9853	12.0219	12.0585	12.0952
12.3164	12.3535	12.3906	12.4278	12.4650
12.6896	12.7272	12.7649	12.8027	12.8405
13.0684	13.1066	13.1448	13.1831	13.2215
13.4527	13.4915	13.4915	13.5691	13.6080
13.8426	13.8819	13.8819	13.9607	14.0002
14.2381	14.2780	14.2780	14.3579	14.3979

Dia.	.0	.1	.2	.3	.4
72	14.4380	14.4781	14.5183	14.5585	14.5988
73	14.8418	14.8825	14.9232	14.9640	15.0049
74	15.2512	15.2925	15.3338	15.3751	15.4165
75	15.6662	15.7080	15.7499	15.7918	15.8337
76	16.0867	16.1291	16.1715	16.2140	16.2565
77	16.5129	16.5558	16.5988	16.6418	16.6849
78	16.9445	16.9880	17.0316	17.0751	17.1188
79	17.3818	17.4258	17.4699	17.5141	17.5583
80	17.8246	17.8692	17.9139	17.9586	18.0033
81	18.2730	18.3182	18.3634	18.4086	18.4540
82	18.7270	18.7727	18.8185	18.8643	18.9102
83	19.1866	19.2328	19.2791	19.3255	19.3719
84	19.6517	19.6985	19.7454	19.7923	19.8393
85	20.1223	20.1697	20.2172	20.2646	20.3122
86	20.5986	20.6465	20.6945	20.7426	20.7907
87	21.0804	21.1289	21.1775	21.2262	21.2747
88	21.5678	21.6169	21.6660	21.7151	21.7643
89	22.0608	22.1104	22.1600	22.2098	22.2595
90	22.5593	22.6095	22.6597	22.7100	22.7603
91	23.0634	23.1141	23.1649	23.2157	23.2666
92	23.5731	23.6244	23.6757	23.7271	23.7785
93	24.0883	24.1402	24.1920	24.2440	24.2960
94	24.6091	24.6615	24.7140	24.7665	24.8190
95	25.1355	25.1885	25.2415	25.2945	25.3376
96	25.6675	25.7210	25.7745	25.8282	25.8818
97	26.2050	26.2591	26.3132	26.3673	26.4216
98	26.7481	26.8027	26.8574	26.9121	26.9669
99	27.2968	27.3519	27.4072	27.4625	27.5178
100	27.8510	27.9067	27.9625	28.0184	28.0743
101	28.4108	28.4671	28.5234	28.5798	28.6363

.5	.6	.7	.8	.9
14.6392	14.6796	14.7201	14.7606	14.8012
15.0458	15.0868	15.1278	15.1689	15.2100
15.4580	15.4950	15.5411	15.5827	15.6244
15.8758	15.9178	15.9600	16.0022	16.0444
16.2991	16.3417	16.3844	16.4272	16.4700
16.7280	16.7712	16.8145	16.8578	16.9011
17.1625	17.2062	17.2500	17.2939	17.3378
17.6025	17.6468	17.6912	17.7356	17.7801
18.0481	18.0930	18.1379	18.1829	18.2280
18.4993	18.5448	18.5902	18.6358	18.6814
18.9561	19.0021	19.0481	19.0942	19.1403
19.4184	19.4650	19.5115	19.5582	19.6049
19.8863	19.9334	19.9806	20.0278	20.0750
20.3598	20.4074	20.4551	20.5029	20.5507
20.8388	20.8870	20.9353	20.9836	21.0320
21.3234	21.3722	21.4210	21.4699	21.5188
21.8136	21.8629	21.9123	21.9617	22.0112
22.3093	22.3592	22.4092	22.4592	22.5092
22.8107	22.8611	22.9116	22.9621	23.0128
23.3176	23.3685	23.4196	23.4707	23.5220
23.8300	23.8816	23.9332	23.9848	24.0367
24.3480	24.4001	24.4523	24.5045	24.5568
24.8716	24.9243	24.9770	25.0298	25.0826
25.4008	25.4540	25.5073	25.5606	25.6140
25.9355	25.9893	26.0432	26.0671	26.1310
26.4759	26.5302	26.5846	26.6090	26.6935
27.0127	27.0766	27.1316	27.1866	27.2416
27.5732	27.6286	27.6841	27.7397	27.7953
28.1302	28.1862	28.2423	28.2984	28.3546
28.6928	28.7494	28.8060	28.8627	28.9194

Dia	.0	.1	.2	.3	.4
102	28.9762	29.0330	29.0899	29.1469	29.2039
103	29.5471	29.6045	29.6620	29.7195	29.7771
104	30.1236	30.1816	30.2396	30.2977	30.3558
105	30.7057	30.7642	30.8228	30.8814	30.9401
106	31.2934	31.3525	31.4116	31.4708	31.5300
107	31.8866	31.9462	32.0059	32.0657	32.1255
108	32.4854	32.5456	32.6058	32.6661	32.7265
109	33.0898	33.1505	33.2113	33.2722	33.3331
110	33.6997	33.7610	33.8224	33.8838	33.9452
111	34.3152	34.3771	34.4390	34.5010	34.5630
112	34.9363	34.9987	35.0612	35.1237	35.1863
113	35.5629	35.6259	35.6889	35.7520	35.8152
114	36.1952	36.2587	36.3223	36.3859	36.4496
115	36.8329	36.8970	36.9612	37.0254	37.0896
116	37.4765	37.5409	37.6056	37.6704	37.7352
117	38.1252	38.1904	38.2557	38.3210	38.3864
118	38.7797	38.8455	38.9113	38.9772	39.0431
119	39.4398	39.5061	39.5725	39.6389	39.7054
120	40.1054	40.1723	40.2392	40.3062	40.3733
121	40.7766	40.8441	40.9116	40.9791	41.0467
122	41.4534	41.5214	41.5895	41.6575	41.7257
123	42.1358	42.2043	42.2729	42.3416	42.4103
124	42.8237	42.8928	42.9619	43.0312	43.1004
125	43.5172	43.5868	43.6566	43.7263	43.7961
126	44.2162	44.2862	44.3567	44.4274	44.4974
127	44.9209	44.9966	45.0625	45.1331	45.2043
128	45.6311	45.7024	45.7738	45.8452	45.9167
129	46.3468	46.4187	46.4907	46.5627	46.6347
130	47.0682	47.1706	47.2721	47.3757	47.4783
131	47.7951	47.8681	47.9412	48.0143	48.0874

.5	.6	.7	.8	.9
29.2610	29.3181	29.3743	29.4325	29.4898
29.8347	29.8924	29.9501	30.0079	30.0657
30.4140	30.4722	30.5305	30.5889	30.6473
30.9989	31.0577	31.1165	31.1754	31.2344
31.5895	31.6487	31.7081	31.7675	31.8270
32.1853	32.2452	32.3052	32.3652	32.4252
32.7865	32.8474	32.9079	32.9685	33.0291
33.3940	33.4551	33.5161	33.5773	33.6385
34.0068	34.0683	34.1300	34.1917	34.2534
34.6251	34.6872	34.7494	34.8116	34.8739
35.2489	35.3116	35.3744	35.4372	35.5000
35.8784	35.9416	36.0049	36.0683	36.1317
36.5134	36.5772	36.6410	36.7049	36.7689
37.1539	37.2183	37.2827	37.3472	37.4117
37.8061	37.8650	37.9300	37.9950	38.0601
38.4518	38.5173	38.5828	38.6484	38.7140
39.1091	39.1751	39.2412	39.3073	39.3735
39.7719	39.8385	39.2052	39.9719	40.0386
40.4403	40.5075	40.5747	40.6420	40.7093
41.1143	41.1820	41.2498	41.3176	41.3855
41.7939	41.8622	41.9305	41.9989	42.0673
42.4790	42.5479	42.6167	42.6858	42.7547
43.1697	43.2391	43.3086	43.3780	43.4476
43.8660	43.9360	44.0059	44.0760	44.1461
44.5679	44.6384	44.7089	44.7795	44.8502
45.2753	45.3463	45.4174	45.4886	45.5598
45.9883	46.0599	46.1312	46.2032	46.2750
46.7068	46.7790	46.8512	46.9235	46.9958
47.4309	47.5037	47.5764	47.6493	47.7222
48.1606	48.2339	48.3073	48.3806	48.4541

Dia	.0	.1	.2	.3	.4
132	48.5276	48.6011	48.6747	48.7484	48.8221
133	49.2656	49.3397	49.4139	49.4881	49.5624
134	50.0093	50.0839	50.1586	50.2334	50.3083
135	50.7584	50.8340	50.9099	50.9842	51.0600
136	51.5132	51.5890	51.6650	51.7410	51.8170
137	52.2740	52.3500	52.4262	52.5030	52.5792
138	53.0394	53.1164	53.1933	53.2703	53.3474
139	53.8109	53.8884	53.9659	54.0434	54.1211
140	54.5880	54.6660	54.7440	54.8222	54.9003
141	55.3706	55.4491	55.5268	55.6064	55.6852
142	56.1587	56.2379	56.3170	56.3963	56.4756
143	56.9525	57.0322	57.1119	57.1917	57.2716
144	57.7518	57.8321	57.9124	57.9927	58.0731
145	58.5567	58.6375	58.7184	58.7993	58.8802
146	59.3672	59.4485	59.5299	59.6114	59.6929
147	60.1832	60.2651	60.3471	60.4291	60.5112
148	61.0048	61.0873	61.1698	61.2524	61.3350
149	61.8320	61.9150	61.9981	62.0812	62.1644
150	62.6648	62.7483	62.8320	62.9157	62.9994
151	63.5030	63.5872	63.6714	63.7556	63.8400
152	64.3470	64.4316	64.5164	64.6012	64.6861
153	65.1964	65.2817	65.3670	65.4523	65.5377
154	66.0514	66.1372	66.2231	66.3090	66.3950
155	66.9120	66.9984	67.0848	67.1713	67.2578
156					

.5	.6	.7	.8	.9
48.8959	48.9697	49.0436	49.1176	49.1916
49.6367	49.7111	49.7856	49.8601	49.9346
50.3832	50.4581	50.5331	50.6082	50.6833
51.1351	51.2110	51.2861	51.3620	51.4374
51.8930	51.9690	52.0450	52.1210	52.1972
52.6560	52.7324	52.8090	52.8860	52.9630
53.4245	53.5017	53.5789	53.6562	53.7335
54.1987	54.2765	54.3543	54.4321	54.5100
54.9786	55.0569	55.1325	55.2136	55.2921
55.7640	55.8428	55.9217	56.0007	56.0797
56.5549	56.6343	56.7138	56.7933	56.8729
57.3515	57.4314	57.5114	57.5915	57.6716
58.1536	58.2341	58.3147	58.3953	58.4760
58.9613	59.0423	59.1235	59.2047	59.2859
59.7745	59.8561	59.9378	60.0196	60.1014
60.5933	60.6755	60.7578	60.8401	60.9224
61.4177	61.5005	61.5833	61.6661	61.7490
62.2477	62.3310	62.4143	62.4978	62.5812
63.0832	63.1671	63.2510	63.3350	63.4190
63.9243	64.0087	64.0932	64.1777	64.2623
64.7710	64.8560	64.9410	65.0261	65.1112
65.6232	65.7088	65.7943	65.8800	65.9657
66.4810	66.5671	66.6533	66.7395	66.8257
67.3444	67.4311	67.5178	67.6045	67.6013

Dia.	.0	.1	.2	.3	.4
1	0.0034	0.0041	0.0049	0.0057	0.0067
2	0.0136	0.0150	0.0165	0.0180	0.0196
3	0.0306	0.0327	0.0348	0.0370	0.0393
4	0.0544	0.0572	0.0600	0.0629	0.0658
5	0.0850	0.0884	0.0919	0.0955	0.0991
6	0.1224	0.1265	0.1307	0.1349	0.1393
7	0.1666	0.1714	0.1763	0.1812	0.1862
8	0.2176	0.2231	0.2286	0.2342	0.2399
9	0.2754	0.2816	0.2878	0.2941	0.3004
10	0.3400	0.3468	0.3537	0.3607	0.3677
11	0.4114	0.4189	0.4265	0.4342	0.4419
12	0.4896	0.4978	0.5061	0.5144	0.5228
13	0.5746	0.5835	0.5924	0.6014	0.6105
14	0.6664	0.6760	0.6856	0.6953	0.7050
15	0.7650	0.7752	0.7855	0.7959	0.8063
16	0.8704	0.8813	0.8923	0.9033	0.9145
17	0.9826	0.9942	1.0059	1.0176	1.0294
18	1.1016	1.1139	1.1262	1.1386	1.1511
19	1.2274	1.2404	1.2534	1.2665	1.2796
20	1.3600	1.3736	1.3873	1.4011	1.4149
21	1.4994	1.5137	1.5281	1.5425	1.5571
22	1.6456	1.6606	1.6757	1.6908	1.7060
23	1.7986	1.8143	1.8300	1.8458	1.8617
24	1.9584	1.9748	1.9912	2.0077	2.0242
25	2.1250	2.1420	2.1591	2.1763	2.1935
26	2.2984	2.3161	2.3339	2.3517	2.3697
27	2.4786	2.4970	2.5155	2.5330	2.5526
28	2.6656	2.6848	2.7028	2.7230	2.7423
29	2.8594	2.8792	2.8990	2.9189	2.9388
30	3.0600	3.0804	3.1009	3.1215	3.1421

.5	.6	.7	.8	.9
0.0077	0.0087	0.0098	0.0110	0.0123
0.0213	0.0230	0.0248	0.0267	0.0286
0.0417	0.0441	0.0465	0.0491	0.0517
0.0689	0.0719	0.0751	0.0781	0.0816
0.1029	0.1066	0.1105	0.1144	0.1184
0.1437	0.1481	0.1526	0.1572	0.1619
0.1913	0.1964	0.2016	0.2069	0.2122
0.2457	0.2515	0.2573	0.2633	0.2693
0.3069	0.3133	0.3199	0.3265	0.3332
0.3748	0.3820	0.3893	0.3966	0.4040
0.4496	0.4575	0.4654	0.4734	0.4815
0.5312	0.5398	0.5484	0.5571	0.5658
0.6196	0.6289	0.6381	0.6475	0.6569
0.7148	0.7247	0.7347	0.7447	0.7548
0.8168	0.8274	0.8381	0.8488	0.8596
0.9256	0.9369	0.9442	0.9596	0.9711
1.0412	1.0532	1.0652	1.0773	1.0894
1.1636	1.1736	1.1889	1.2017	1.2145
1.2928	1.3061	1.3195	1.3329	1.3464
1.4288	1.4428	1.4569	1.4710	1.4852
1.5716	1.5863	1.6010	1.6158	1.6307
1.7212	1.7366	1.7520	1.7675	1.7830
1.8776	1.8937	1.9097	1.9259	1.9421
2.0408	2.0575	2.0743	2.0911	2.1080
2.2108	2.2282	2.2457	2.2632	2.2808
2.3876	2.4057	2.4238	2.4420	2.4603
2.5712	2.5900	2.6088	2.6277	2.6466
2.7616	2.7811	2.8005	2.8201	2.8396
2.9588	2.9789	2.9991	3.0193	3.0396
3.1628	3.1836	3.2045	3.2254	3.2464

Dia.	.0	.1	.2	.3	.4
31	3.2674	3.2885	3.3097	3.3309	3.3523
32	3.4816	3.5034	3.5253	3.5472	3.5692
33	3.7026	3.7251	3.7476	3.7702	3.7929
34	3.9304	3.9536	3.9768	4.0001	4.0234
35	4.1650	4.1888	4.2127	4.2367	4.2607
36	4.4064	4.4309	4.4555	4.4801	4.5049
37	4.6546	4.6798	4.7051	4.7304	4.7558
38	4.9096	4.9355	4.9614	4.9874	5.0135
39	5.1714	5.1980	5.2246	5.2513	5.2780
40	5.4400	5.4672	5.4945	5.5219	5.5493
41	5.7154	5.7433	5.7713	5.7993	5.8275
42	5.9976	6.0262	6.0549	6.0836	6.1124
43	6.2866	6.3159	6.3452	6.3746	6.4041
44	6.5824	6.6124	6.6424	6.6725	6.7026
45	6.8850	6.9156	6.9463	6.9771	7.0079
46	7.1944	7.2257	7.2571	7.2885	7.3201
47	7.5106	7.5426	7.5747	7.6068	7.6390
48	7.8336	7.8663	7.8990	7.9318	7.9647
49	8.1634	8.1968	8.2302	8.2637	8.2972
50	8.5000	8.5340	8.5681	8.6023	8.6365
51	8.8434	8.8781	8.9129	8.9477	8.9827
52	9.1936	9.2290	9.2645	9.3000	9.3356
53	9.5506	9.5867	9.6228	9.6590	9.6953
54	9.9144	9.9512	9.9880	10.0249	10.0618
55	10.2850	10.3224	10.3599	10.3975	10.4351
56	10.6624	10.7005	10.7387	10.7765	10.8153
57	11.0466	11.0854	11.1243	11.1632	11.2022
58	11.4370	11.4771	11.5166	11.5562	11.5959
59	11.8354	11.8756	11.9158	11.9561	11.9964
60	12.2400	12.2808	12.3217	12.3627	12.4037

Circles Area's in Wine-Gallons. 169

5.	6.	7.	8.	9.
3.3736	3.3951	3.4166	3.4382	3.4599
3.5912	3.6134	3.6356	3.6579	3.6802
3.8156	3.8385	3.8613	3.8843	3.9073
4.0468	4.0703	4.0939	4.1175	4.1412
4.2848	4.3090	4.3333	4.3576	4.3820
4.5296	4.5545	4.5794	4.6044	4.6295
4.7812	4.8068	4.8324	4.8581	4.8838
5.0396	5.0659	5.0921	5.1185	5.1449
5.3048	5.3317	5.3587	5.3857	5.4128
5.5768	5.6044	5.6321	5.6598	5.6876
5.8556	5.8839	5.9122	5.9406	5.9691
6.1412	6.1702	6.1992	6.2283	6.2574
6.4336	6.4633	6.4929	6.5227	6.5525
6.7328	6.7631	6.7935	6.8239	6.8544
7.0388	7.0698	7.1009	7.1320	7.1632
7.3516	7.3833	7.4150	7.4468	7.4787
7.6712	7.7036	7.7360	7.7685	7.8010
7.9976	8.0307	8.0637	8.0969	8.1301
8.3308	8.3645	8.3983	8.4321	8.4660
8.6708	8.7052	8.7397	8.7742	8.8088
9.0176	9.0527	9.0878	9.1230	9.1583
9.3712	9.4070	9.4428	9.4787	9.5146
9.7316	9.7681	9.8045	9.8411	9.8777
10.0988	10.1359	10.1731	10.2103	10.2476
10.4728	10.5106	10.5485	10.5864	10.6244
10.8536	10.8921	10.9306	10.9692	11.0079
11.2412	11.2804	11.3196	11.3589	11.3982
11.6356	11.6755	11.7153	11.7553	11.7953
12.0368	12.0773	12.1179	12.1585	12.1992
12.4448	12.4860	12.5273	12.5686	12.6100

THE foregoing Table is useful in Gauging of Wine-Vessels.

Example. There is a *Brandy Butt* supposed to be in the form of the middle Frustum of a *Spheroid*: The Bung-Diameter is 32 Inches, the Head-Diameter 24 Inches, and the Length 40 Inches. What is the Content of this Cask in Wine-Gallons?

From the Bung-Diameter ————— 32
Subtract the Head-Diameter ————— 24
The Difference is ————— 8

TABLES
OF
EXCISE,

According to the
Respective Allowances

Made to
Common Brewers,

Both in
L O N D O N

And the
COUNTRY.

Excise for Common Brewers in the Country at 5 s. per Barrel, the Allowance being $2\frac{1}{2}$ Barrels in every 23.

	0					$\frac{1}{4}$					$\frac{1}{2}$					$\frac{3}{4}$				
B	l.	s.	d.	f.	23	l.	s.	d.	f.	23	l.	s.	d.	f.	23	l.	s.	d.	f.	23
1	0	4	5	1	21	0	5	6	3	9	0	6	8	0	20	0	7	9	2	8
2	0	8	10	3	19	0	10	0	1	7	0	11	1	2	18	0	12	3	0	6
3	0	13	4	1	17	0	14	5	3	5	0	15	7	0	16	0	16	8	2	4
4	0	17	9	3	15	0	18	11	1	3	1	0	0	2	14	1	1	2	0	2
5	1	2	3	1	13	1	3	4	3	1	1	4	6	0	12	1	5	7	2	0
6	1	6	8	3	11	1	7	10	9	22	1	8	11	2	10	1	10	9	3	31
7	1	11	2	1	9	1	12	3	2	20	1	13	5	0	8	1	14	6	1	19
8	1	15	7	3	7	1	16	9	0	18	1	17	10	2	6	1	18	11	3	17
9	2	0	1	1	5	2	1	2	2	16	2	2	4	0	4	2	3	5	1	15
10	2	4	6	3	3	2	5	8	0	14	2	6	9	2	2	2	7	10	3	13
11	2	9	0	1	1	2	10	1	2	12	2	11	3	0	0	2	12	4	1	11
12	2	13	5	2	22	2	14	7	0	10	2	15	8	1	21	2	16	9	3	9
13	2	17	11	0	20	2	19	0	2	8	3	0	1	3	19	3	1	3	1	7
14	3	2	4	2	18	3	3	6	0	6	3	4	7	1	17	3	5	8	3	5
15	3	6	10	0	16	3	7	1	2	4	3	9	0	3	15	3	10	2	1	3
16	3	11	5	2	14	3	12	5	0	2	3	13	6	1	13	3	14	7	3	1
17	3	15	9	0	12	3	16	10	2	0	3	17	11	3	11	3	19	1	0	32
18	4	0	2	2	10	4	1	3	3	21	4	2	5	1	9	4	3	6	2	20
19	4	4	8	0	8	4	5	9	1	19	4	6	10	3	7	4	8	0	0	18
20	4	9	12	6		4	10	2	3	17	4	11	4	2	5	4	12	5	2	16
21	4	13	7	0	4	4	14	8	1	15	4	15	9	3	3	4	16	11	0	14
22	4	18	0	2	2	4	19	1	3	13	5	0	3	1	15	5	1	4	2	12

Excise

Excise for Common Brewers in the County at
5 s. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
23	5	2	6	0	713	158	17	6	0
46	10	5	0	0	736	164	0	0	0
69	15	7	6	0	759	169	2	6	0
92	20	10	0	0	782	174	5	0	0
115	25	12	6	0	805	179	7	6	0
138	30	15	0	0	828	184	10	0	0
161	35	17	6	0	851	189	12	6	0
184	41	0	0	0	874	194	15	0	0
207	46	2	6	0	897	199	17	6	0
230	51	5	0	0	920	205	0	0	0
253	56	7	6	0	943	210	2	6	0
276	61	10	0	0	966	215	5	0	0
299	66	12	6	0	989	220	7	6	0
322	71	15	0	0	1012	225	10	0	0
345	76	17	6	0	1035	230	12	6	0
368	82	0	0	0	1058	235	15	0	0
391	87	2	6	0	1081	240	17	6	0
414	92	5	0	0	1104	246	0	0	0
437	97	7	6	0	1127	251	2	6	0
460	102	10	0	0	1150	256	5	0	0
483	107	12	6	0	1173	261	7	6	0
506	112	15	0	0	1196	266	10	0	0
529	117	17	6	0	1219	271	12	6	0
552	123	0	0	0	1242	276	15	0	0
575	128	2	6	0	1265	281	17	6	0
598	133	5	0	0	1288	287	0	0	0
621	138	7	6	0	1311	292	2	6	0
644	143	10	0	0	1334	297	5	0	0
667	148	12	6	0	1357	302	7	6	0
690	153	15	0	0	1380	307	10	0	0

Excise for Common Brewers in the Coun-
try at 5 s. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
1403	312	12	6	0	2093	466	7	6	0
1426	317	15	0	0	2116	471	10	0	0
1449	322	17	6	0	2139	476	12	6	0
1472	328	0	0	0	2162	481	15	0	0
1495	333	2	6	0	2185	486	17	6	0
1518	338	5	0	0	2208	492	0	0	0
1541	343	7	6	0	2231	497	2	6	0
1564	348	10	0	0	2254	502	5	0	0
1587	353	12	6	0	2277	507	7	6	0
1610	358	15	0	0	2300	512	10	0	0
1633	363	17	6	0	2323	517	12	6	0
1656	369	0	0	0	2346	522	15	0	0
1679	374	2	6	0	2369	527	17	6	0
1702	379	5	0	0	2392	533	0	0	0
1725	384	7	6	0	2415	538	2	6	0
1748	389	10	0	0	2438	543	5	0	0
1771	394	12	6	0	2461	548	7	6	0
1794	399	15	0	0	2484	553	10	0	0
1817	404	17	6	0	2507	558	12	6	0
1840	410	0	0	0	2530	563	15	0	0
1863	415	2	6	0	5060	1127	10	0	0
1886	420	5	0	0	7590	1691	5	0	0
1909	425	7	6	0	10120	2255	0	0	0
1932	430	10	0	0	12650	2818	15	0	0
1955	435	12	6	0	15180	3382	10	0	0
1978	440	15	0	0	17710	3946	5	0	0
2001	445	17	6	0	2040	4510	0	0	0
2024	451	0	0	0	22770	5073	15	0	0
2047	456	2	6	0	25300	5637	10	0	0
2070	461	5	0	0	27830	6201	5	0	0

Excise

Excise for Common Brewers in the Country, at 1 s. 4 d. per Barrel, the Allowances being $2\frac{1}{2}$ Barrels in every 23.

B.	0					$\frac{1}{2}$					$\frac{1}{3}$					$\frac{1}{4}$				
	l.	s.	d.	f.	23	l.	s.	d.	f.	23	l.	s.	d.	f.	23	l.	s.	d.	f.	23
1	0	1	2	1	1	0	1	5	3	7	0	1	9	1	13	0	2	0	3	19
2	0	2	4	2	2	0	2	8	0	8	0	2	11	2	14	0	3	3	0	20
3	0	3	6	3	3	0	3	10	1	9	0	4	1	3	15	0	4	5	1	21
4	0	4	9	0	4	0	5	0	2	10	0	5	4	0	16	0	5	7	2	22
5	0	5	11	1	5	0	6	2	3	11	0	6	6	1	17	0	6	10	0	0
6	0	7	1	2	6	0	7	5	0	12	0	7	8	2	18	0	8	0	1	1
7	0	8	3	3	7	0	8	7	1	13	0	8	10	3	19	0	9	2	2	2
8	0	9	6	0	8	0	9	9	2	14	0	10	1	0	20	0	10	4	3	3
9	0	10	8	1	9	0	10	11	3	15	0	11	3	1	21	0	11	7	0	4
10	0	11	10	2	10	0	12	2	0	16	0	12	5	2	22	0	12	9	1	5
11	0	13	0	3	11	0	13	4	1	17	0	13	8	0	0	0	13	11	2	6
12	0	14	3	0	12	0	14	6	2	18	0	14	10	1	1	0	15	1	3	7
13	0	15	5	1	13	0	15	8	3	19	0	16	0	2	2	0	16	4	0	8
14	0	16	7	2	14	0	16	11	0	20	0	17	2	3	3	0	17	6	1	9
15	0	17	9	3	15	0	18	1	1	21	0	18	5	0	4	0	18	8	2	10
16	0	19	0	0	16	0	19	3	2	22	0	19	7	1	5	0	19	10	3	11
17	1	0	2	1	17	1	0	6	0	0	1	0	9	2	6	1	1	1	0	12
18	1	1	4	2	18	1	1	8	1	1	1	1	11	3	7	1	2	3	1	13
19	1	2	6	3	19	1	2	10	2	2	1	2	2	0	8	1	3	5	2	14
20	1	3	9	0	20	1	4	0	3	3	1	4	4	1	9	1	4	7	3	15
21	1	4	11	1	21	1	5	3	0	4	1	5	6	2	10	1	5	10	0	16
22	1	6	1	2	22	1	6	5	1	5	1	6	8	3	11	1	7	0	1	17

Excise for Common Brewers in the County of
1 s. 4 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
23	1	7	4	0	713	42	7	4	0
46	2	4	8	0	736	43	14	8	0
69	4	2	0	0	759	45	2	0	0
92	5	9	4	0	782	46	9	4	0
115	6	16	8	0	805	47	16	8	0
138	8	4	0	0	828	49	4	0	0
161	9	11	4	0	851	50	11	4	0
184	10	18	8	0	874	51	18	8	0
207	12	6	0	0	897	53	6	0	0
230	13	13	4	0	920	54	13	4	0
253	15	0	8	0	943	56	0	8	0
276	16	8	0	0	966	57	8	0	0
299	17	15	4	0	989	58	15	4	0
322	19	2	8	0	1012	60	2	8	0
345	20	10	0	0	1035	61	10	0	0
368	21	17	4	0	1058	62	17	4	0
391	23	4	8	0	1081	64	4	8	0
414	24	12	0	0	1104	65	12	0	0
437	25	19	4	0	1127	66	19	4	0
460	27	6	8	0	1150	68	6	8	0
483	28	14	0	0	1173	69	14	0	0
506	30	1	4	0	1196	71	1	4	0
529	31	8	8	0	1219	72	8	8	0
552	32	16	0	0	1242	73	16	0	0
575	34	3	4	0	1265	75	3	4	0
598	35	10	8	0	1288	76	10	8	0
621	36	18	0	0	1311	77	18	0	0
644	38	5	4	0	1334	79	5	4	0
667	39	12	8	0	1357	80	12	8	0
690	41	0	0	0	1380	82	0	0	0

Excise for Common Brewers in the County, at
1 s. 4 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
1403	83	7	4	0	209	124	7	4	0
1426	84	14	8	0	2116	125	14	8	0
1449	86	2	0	0	2139	127	2	0	0
147	87	9	4	0	2162	128	9	4	0
1495	88	16	8	0	2185	129	16	8	0
1518	90	4	0	0	2208	131	4	0	0
1541	91	11	4	0	2231	132	11	4	0
1564	92	18	8	0	2254	133	18	8	0
1587	94	6	0	0	2277	135	6	0	0
1610	95	13	4	0	2300	136	13	4	0
1633	97	0	8	0	2323	138	0	8	0
1656	98	8	0	0	2346	139	8	0	0
1679	99	15	4	0	2369	140	15	4	0
1702	101	2	8	0	2392	142	2	8	0
1725	102	10	0	0	2415	143	10	0	0
1748	103	17	4	0	2438	144	17	4	0
1771	105	4	8	0	2461	146	4	8	0
1794	106	12	0	0	2484	147	12	0	0
1817	107	19	4	0	2507	148	19	4	0
1840	109	6	8	0	2530	150	6	8	0
1863	110	14	0	0	5060	300	13	4	0
1886	112	1	4	0	7590	451	0	0	0
1909	113	8	8	0	10120	601	6	8	0
1932	114	16	0	0	12650	751	13	4	0
1955	116	3	4	0	15180	902	0	0	0
1978	117	10	8	0	17710	1052	6	8	0
2001	118	18	0	0	20240	1202	13	4	0
2024	120	5	4	0	22770	1353	0	0	0
2047	121	12	8	0	25300	1503	6	8	0
2070	123	0	0	0	27830	1653	13	4	0

Excise for Common Brewers in London, at
5 s. per Barrel, the Allowances being 3
Barrels in every 23 Beer.

B.	0				$\frac{1}{4}$				$\frac{1}{2}$				$\frac{3}{4}$							
	l.	s.	d.	f.	23	l.	s.	d.	f.	23	l.	s.	d.	f.	23	l.	s.	d.	f.	23
1	0	4	4	0	16	0	5	5	0	20	0	6	6	1	10	0	7	7	1	5
2	0	8	8	1	9	0	9	9	1	13	0	10	10	1	17	0	11	11	1	21
3	0	13	0	2	2	1	14	1	2	6	0	15	2	2	10	0	16	3	2	14
4	0	17	4	2	18	1	18	5	2	22	0	19	6	3	3	1	0	7	3	7
5	1	1	8	3	11	1	2	9	3	15	1	3	10	3	19	1	5	0	0	0
6	1	6	1	0	4	1	7	2	0	8	1	8	3	0	12	1	9	4	0	16
7	1	10	5	0	20	1	13	6	1	11	1	12	7	1	5	1	13	8	1	5
8	1	14	9	1	13	1	15	10	1	17	2	16	11	1	21	1	18	0	2	2
9	1	19	1	2	6	2	0	2	2	10	2	1	3	2	1	2	2	4	2	16
10	2	3	5	2	22	2	4	6	3	3	2	5	7	3	7	2	6	8	3	11
11	2	7	9	3	15	2	8	10	3	19	2	10	0	0	0	2	11	1	0	4
12	2	12	2	0	8	2	13	3	0	12	2	14	4	0	16	2	15	3	0	2
13	2	16	6	1	1	2	17	7	1	5	3	18	8	1	9	2	19	9	1	13
14	3	0	10	1	17	3	1	11	1	21	3	3	0	2	23	4	1	2	6	6
15	3	5	2	2	10	3	6	3	2	14	3	7	4	2	18	3	8	5	3	23
16	3	9	6	3	3	3	10	7	3	7	3	11	8	3	11	3	12	9	3	13
17	3	13	10	3	19	3	15	0	0	0	4	16	1	0	4	3	17	2	0	8
18	3	18	3	0	12	3	19	4	0	16	4	0	5	0	20	4	1	6	1	3
19	4	2	7	1	5	3	3	8	1	9	4	4	9	1	13	4	5	10	1	17
20	4	6	11	1	21	4	8	0	2	24	9	1	2	6	4	10	2	2	10	10
21	4	11	3	2	14	4	12	4	2	18	4	13	5	2	22	4	14	6	3	3
22	4	15	7	3	7	4	16	8	3	11	4	17	9	3	15	4	18	10	3	19

Excise

Excise for Common Brewers in London, at
5 s. per Barrel, &c. Beer.

Bar.	l.	Bar.	l.	Bar.	l.	Bar.	l.
23	5	713	155	1403	305	2093	455
46	10	736	160	1426	310	2116	460
69	15	759	165	1449	315	2139	465
92	20	782	170	1472	320	2162	470
115	25	805	175	1495	325	2185	475
138	30	828	180	1518	330	2208	480
161	35	851	185	1541	335	2231	485
184	40	874	190	1564	340	2254	490
207	45	897	195	1587	345	2277	495
230	50	920	200	1610	350	2300	500
253	55	943	205	1633	355	2323	505
276	60	966	210	1656	360	2346	510
299	65	989	215	1679	365	2369	515
322	70	1012	220	1702	370	2392	520
345	75	1035	225	1725	375	2415	525
368	80	1058	230	1748	380	2439	530
391	85	1081	235	1771	385	2461	535
414	90	1104	240	1794	390	2485	540
437	95	1127	245	1817	395	2507	545
460	100	1150	250	1840	400	2530	550
483	105	1173	255	1863	405	3060	1100
506	110	1196	260	1886	410	7590	1650
529	115	1219	265	1909	415	10120	2200
552	120	1242	270	1932	420	12650	2750
575	125	1265	275	1955	425	15180	3300
598	130	1288	280	1978	430	17710	3850
621	135	1311	285	2001	435	20240	4400
644	140	1334	290	2024	440	22770	4950
667	145	1357	295	2047	445	25300	5500
690	150	1380	300	2070	450	27830	6050

Excise

Excise for Common Brewers in London,
at 5 s. per Barrel, the Allowances being
2 Barrels in every 22 of Ale.

B.	0				$\frac{1}{2}$				$\frac{1}{2}$				$\frac{3}{4}$			
	l.	s.	d.	f.	l.	s.	d.	f.	l.	s.	d.	f.	l.	s.	d.	f.
1	0	4	6	2	4	0	5	8	0	16	0	6	9	3	6	0
2	0	9	1	0	8	0	10	2	2	20	6	11	4	1	10	0
3	0	13	7	2	12	0	14	9	1	2	0	15	10	3	14	0
4	0	18	2	0	16	0	19	3	3	6	1	0	5	1	18	0
5	1	2	8	2	20	1	3	10	1	10	1	3	0	0	9	1
6	1	7	3	1	4	1	8	4	3	14	1	9	6	2	4	1
7	1	11	9	3	6	1	12	11	1	18	1	14	1	0	8	1
8	1	16	4	1	10	1	17	6	0	0	1	18	7	2	12	1
9	2	0	10	3	14	2	2	0	2	4	2	3	2	0	16	2
10	2	5	5	1	18	2	6	7	0	8	2	7	8	0	20	2
11	2	10	0	0	0	2	11	1	2	12	3	12	3	1	2	2
12	2	14	6	2	4	2	15	8	0	16	2	16	9	3	6	2
13	2	19	1	0	8	3	0	2	2	20	3	1	4	1	10	3
14	3	3	7	2	12	3	4	9	1	2	3	5	10	3	14	3
15	3	8	2	0	16	3	9	3	3	6	3	10	5	1	18	3
16	3	12	8	2	20	3	13	10	1	10	3	15	0	0	4	3
17	3	17	3	1	4	3	18	4	3	14	3	19	6	2	4	4
18	4	1	9	3	6	4	2	11	1	18	4	4	1	0	8	4
19	4	6	4	1	10	4	7	6	0	0	4	8	7	2	12	4
20	4	10	10	3	14	4	12	0	2	4	4	13	2	0	16	4
21	4	15	5	1	18	4	16	7	0	8	4	17	8	2	20	4

Excise

Excise for Common Brewers in London at 5s. per Barrel, &c. Ale.

Bar.	L.	Bar.	L.	Bar.	L.	Bar.	L.
22	5	682	155	1342	305	2002	455
44	10	704	160	1364	310	2024	460
66	15	726	165	1386	315	2046	465
88	20	748	170	1408	320	2068	470
110	25	770	175	1430	325	2090	475
132	30	792	180	1452	330	2112	480
154	35	814	185	1474	335	2134	485
176	40	836	190	1496	340	2156	490
198	45	858	195	1518	345	2178	495
220	50	880	200	1540	350	2200	500
242	55	902	205	1562	355	2222	505
264	60	924	210	1584	360	2244	510
286	65	946	215	1606	365	2266	515
308	70	968	220	1628	370	2288	520
330	75	990	225	1650	375	2310	525
352	80	1012	230	1672	380	2332	530
374	85	1034	235	1694	385	2354	535
396	90	1056	240	1716	390	2376	540
418	95	1078	245	1738	395	2398	545
440	100	1100	250	1760	400	2420	550
462	105	1122	255	1782	405	2442	555
484	110	1144	260	1804	410	2464	560
506	115	1166	265	1826	415	2486	565
528	120	1188	270	1848	420	2508	570
550	125	1210	275	1870	425	2530	575
572	130	1232	280	1892	430	2552	580
594	135	1254	285	1914	435	2574	585
616	140	1276	290	1936	440	2596	590
638	145	1298	295	1958	445	2618	595
660	150	1320	300	1980	450	2640	600

Excise

Excise for Common Brewers in *London*, at
1 s. 4 d. per Barrel, the Allowances be-
ing 3 Barrels in every 23 *Small Beer*.

B.	0				l.	$\frac{1}{2}$				l.	$\frac{1}{3}$				l.	$\frac{1}{4}$			
	s.	d.	f.	23		s.	d.	f.	23		s.	d.	f.	23		s.	d.	f.	23
10	1	1	3	15	0	1	5	1	13	0	1	8	3	11	0	2	0	1	9
20	2	3	3	7	0	2	7	1	5	0	2	10	3	3	0	3	2	1	1
30	3	5	2	22	0	3	9	0	20	0	4	0	2	18	0	4	4	0	16
40	4	7	2	14	0	4	11	0	12	0	5	2	2	10	0	5	6	0	8
50	5	9	2	6	0	6	1	0	4	0	6	4	2	2	0	6	8	0	0
60	6	11	1	21	0	7	2	3	19	0	7	6	1	17	0	7	9	3	15
70	8	1	1	13	0	8	4	3	11	0	8	8	1	9	0	8	11	3	7
80	9	3	1	5	0	9	6	3	3	0	9	10	1	1	0	10	1	2	23
90	10	5	1	20	0	10	8	2	18	0	11	0	0	16	0	11	3	2	14
100	11	7	0	12	0	11	10	2	10	0	12	2	0	8	0	12	5	2	6
110	12	9	0	4	0	13	0	2	2	0	13	4	0	0	0	13	7	1	21
120	13	10	3	39	0	14	2	1	17	0	14	5	3	15	0	14	9	1	13
130	15	0	3	11	0	15	4	1	9	0	15	7	3	7	0	15	11	1	5
140	16	2	3	3	0	16	6	1	1	0	16	9	2	22	0	17	1	0	20
150	17	4	0	16	0	17	8	0	16	0	17	11	2	14	0	18	3	0	12
160	18	6	2	10	0	18	10	0	8	0	19	1	2	6	0	19	5	0	4
170	19	8	2	2	1	0	0	0	0	1	0	3	1	21	1	0	6	3	19
180	0	10	1	17	1	1	1	3	15	1	1	5	1	13	1	1	8	3	11
190	2	0	1	9	1	2	3	3	7	1	2	7	1	5	1	2	10	3	3
200	3	2	1	1	1	3	5	2	22	1	3	9	0	20	1	4	0	2	18
210	4	4	0	6	1	4	7	2	14	1	4	11	0	12	1	5	2	2	10
220	5	6	0	8	1	5	9	2	6	1	6	1	0	4	1	6	4	2	2

Excise

Excise for Common Brewers in London, at
1s. 4d. per Barrel, &c. Small Beer.

Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.
23	1	6	8	713	41	6	8	1403	81	6	8	2093	131	6	8				
46	2	13	4	536	42	13	4	1426	82	13	4	2116	122	13	4				
69	4	0	0	759	44	0	0	1449	84	0	0	2139	124	0	0				
92	5	6	8	782	45	6	8	1472	85	6	8	2162	125	6	8				
115	6	13	4	805	46	13	4	1495	86	13	4	2185	126	13	4				
138	8	0	0	828	48	0	0	1518	88	0	0	2208	128	0	0				
161	9	6	8	851	49	6	8	1541	89	6	8	2231	129	6	8				
184	10	13	4	874	50	13	4	1564	90	13	4	2254	130	13	4				
207	12	0	0	897	52	0	0	1587	92	0	0	2277	132	0	0				
230	13	6	8	920	53	6	8	1610	93	6	8	2300	133	6	8				
253	14	13	4	943	54	13	4	1633	94	13	4	2323	134	13	4				
276	16	0	0	966	56	0	0	1656	96	0	0	2346	136	0	0				
299	17	6	8	989	57	6	8	1679	97	6	8	2369	137	6	8				
322	18	13	4	1012	58	13	4	1702	98	13	4	2392	138	13	4				
345	20	0	0	1035	60	0	0	1725	100	0	0	2415	140	0	0				
368	21	6	8	1058	61	6	8	1748	101	6	8	2438	141	6	8				
391	22	13	4	1081	62	13	4	1771	102	13	4	2461	142	13	4				
414	24	0	0	1104	64	0	0	1794	104	0	0	2484	144	0	0				
437	25	6	8	1127	65	6	8	1817	105	6	8	2507	145	6	8				
460	26	13	4	1150	66	13	4	1840	106	13	4	2530	146	13	4				
483	28	0	0	1173	68	0	0	1863	108	0	0	5060	293	6	8				
506	29	6	8	1196	69	6	8	1886	109	6	8	7590	440	0	0				
529	30	13	4	1219	70	13	4	1909	110	13	4	10120	586	13	4				
552	32	0	0	1242	72	0	0	1932	112	0	0	12650	733	6	8				
575	33	6	8	1265	73	6	8	1955	113	6	8	15180	880	0	0				
598	34	13	4	1288	74	13	4	1978	114	13	4	17710	1026	13	4				
621	36	0	0	1311	76	0	0	2001	116	0	0	20240	1173	6	8				
644	37	6	8	1334	77	6	8	2024	117	6	8	22770	1320	0	0				
667	36	13	4	1357	78	13	4	2047	118	13	4	25300	1466	13	4				
690	40	0	0	1380	80	0	0	2070	120	0	0	27830	1613	6	8				

Excise for Victuallers in the Country at 5 s. per
Barrel, Strong Beer, and Ale :

Bar	l.	s.	d.	ar	l.	s.	d.
1	0	1	3	28	7	0	0
2	0	2	6	29	7	5	0
3	0	3	9	30	7	10	0
4	0	5	0	31	7	15	0
5	0	10	0	32	8	0	0
6	0	15	0	33	8	5	0
7	1	0	0	34	8	10	0
8	1	5	0	35	8	15	0
9	1	10	0	36	9	0	0
10	1	15	0	37	9	5	0
11	2	0	0	38	9	10	0
12	2	5	0	39	9	15	0
13	2	10	0	40	10	0	0
14	2	15	0	50	12	10	0
15	3	0	0	60	15	0	0
16	3	5	0	70	17	10	0
17	3	10	0	80	20	0	0
18	3	15	0	90	22	10	0
19	4	0	0	100	25	0	0
20	4	5	0	110	27	10	0
21	4	10	0	120	30	0	0
22	4	15	0	130	32	10	0
23	5	0	0	140	35	0	0
24	5	5	0	150	37	10	0
25	5	10	0	160	40	0	0
26	5	15	0	170	42	10	0
27	6	0	0	180	45	0	0
28	6	5	0	190	47	10	0
29	6	10	0	200	50	0	0
30	6	15	0				

Excise for Victuallers in the Country, at 1 s. 4 d.
per Barrel Small Beer.

Bar.	l.	s.	d.	Bar.	l.	s.	d.
1	0	0	4	28	1	1	4
2	0	0	8	29	1	18	8
3	0	1	0	30	2	0	0
4	0	1	4	31	2	1	4
5	0	2	8	32	2	2	8
6	0	4	0	33	2	4	0
7	0	5	4	34	2	5	4
8	0	6	8	35	2	6	8
9	0	8	0	36	2	8	0
10	0	9	4	37	2	9	4
11	0	10	8	38	2	10	8
12	0	12	0	39	2	12	0
13	0	13	4	40	2	13	4
14	0	14	8	50	3	6	8
15	0	16	0	60	4	0	0
16	0	17	4	70	4	13	4
17	0	18	8	80	5	6	8
18	1	0	0	90	6	0	0
19	1	1	4	100	6	13	4
20	1	2	8	110	7	6	8
21	1	4	0	120	8	0	0
22	1	5	4	130	8	13	4
23	1	6	8	140	9	6	8
24	1	8	0	150	10	0	0
25	1	9	4	160	10	13	4
26	1	10	8	70	11	6	0
27	1	12	0	180	12	0	0
28	1	13	4	190	12	13	4
29	1	14	8	200	13	6	8
30	1	16	0				

The Use of the Tables of

The foregoing Tables (though very short) will shew the exact Duty for any Number of Barrels; and their Use will be render'd very plain and easie, by the Directions and Example following.

1. If the number of Barrels of strong or small Beer given be less than 23, seek it in the first Column of the Table, and against it you have the exact Duty in Pounds, Shillings, Pence, Farthings, and 23d parts of a Farthing. Thus by the first Table the Duty of 10 Barrels and a half will be found to be 2*l.* 6*s.* 9*d.* 2*f.* and 2 parts of a Farthing; for against 10 in the Column of Barrels, under $\frac{1}{2}$ you have this Sum.

2. For any Number above 23, seek it in some of the Columns of Barrels in the Table, and if you cannot find the same Number, take the Next less, and set down the Sum of Money against it; then subtract the Number found from the Number given, the Remainder (which will ever be less than 23) being found in the first Column of the Table, gives you a Sum of Money, which added to the former, is the exact Duty required.

8	0	7	
0	0	2	0
4	0	8	0
0	0	9	1
0	0	0	1
7	0	0	1
0	0	1	0
0	0	4	0
4	0	1	0
0	0	1	0

0	1	1	1
4	1	1	0
0	0	1	0
0	0	1	1
4	0	1	1
0	0	1	1
0	0	1	1
0	0	1	1
0	0	1	1
0	0	1	1

Example.

Example.

Allowances for Common Brewers. 187

Example.

*What Sum of Money will the Duty of 1306½
Barrels of Strong Beer amount unto?*

See the Work by the first Table.

Number given	1306½	l.	s.	d.	f.
Next less per Table	1288	287	0	0	0
Remainder	18½	4	2	5	19

The exact Duty is 291 : 2 : 5 : 1 : 9
That is 291 l. 2 s. 5 d. 1 f. 9 parts of a Far-
thing.

And so for any other Number of Barrels by this
or any of the other Tables.

SECT.

 S E C T. XI.

 Short R U L E S
 F O R

Gauging of Malt.

THE Duty upon Malt is charged upon the Bushel, and so proportionably for a greater or lesser Quantity. The Bushel here intended is the *Winchester-Bushel*, which (as settled by the first Act for laying a Duty upon Malt) contains 2150.42, solid Inches, but in Practice 2150, will be sufficiently near the Truth.

The Area of any plain Figure may be found in Inches, by the Rules laid down in *Section III.* Now if the Area in Inches be divided by 2150, the Quotient shews the Bushels and Parts which such Figure or Vessel contains at one Inch deep.

Example. There is a *Cistern* or *Uting* *Fat*, whose Base is a *Rectangular Parallelogram*, the Length is 72 Inches, and the Breadth 48 Inches, what is the Area or Content at one Inch deep? *Answer*, 1.607, that is, one Bushel, and .607 thousand Parts of a Bushel. For 72 multiplied by 48, is 3456; and this divided by 2150, gives 1.607. But

Sect. XI. Gauging of Malt. 189

But the Area of any such Figure may be more readily found by the Lines *A* and *B* upon the Rule. Thus,

As 2150 upon *A*, is to one of the sides upon *B*;
So is the other side upon *A*, to the Area upon *B*.

So in the Example above :

Set 2150 upon *A*, to 71 upon *B*;
Then against 48 upon *A*, is 1.607 upon *B*;
Or let 2150 upon *A*, to 48 upon *B*;
Then against 72 upon *A*, is 1.607 upon *B*.

The Area thus found, being multiplied by the Mean Depth in Inches, gives the Content of such Cistern at that Depth ; and so for any quantity of Malt upon the Floor. But care must be taken in finding the Mean Depth of any quantity of Malt, either in the Cistern or upon the Floor, by reason of the Unevenness of the Surface of the Malt, and of the bottom of the Cistern or Floor upon which it is laid ; in order to which, take the Depth in 6, 8, or 10 Places ; add all these Depths together, and divide the Sum by the number of Places in which the Depths were taken, the Quotient will be the Mean Depth required.

Example. Suppose a Quantity of Malt upon a Floor, of which you take the Depth in Eight several Places, and find,

The

The	<	4	>	Depth	to be	<	Inches.

The Sum of these is 40.8, which divided by 8, (the Number of places) gives 5.1 for the Mean Depth.

Barley is sometimes steep'd in Round Vessels, and the Area of a Circle in Bushels is thus found: Take the Diameter in Inches, and let the Square thereof be multiplied by .0003643, or divided by 2737.47, the Product or Quotient will be the Area sought.

Example. Suppose the Diameter be 60 Inches, this square is 3600, and this multiplied by .0003653, or divided by 2737.47, gives 1.315, the Area required.

But for any Round Vessel, the Rule gives the Area in Bushels by Inspection, by help of a certain Number call'd the *Gage-Point*, which in this case is 52.32, (being the Diameter of a Circle, whose Area is 2150, the number of solid Inches contain'd in the Bushel.) Thus in the Example above, the Diameter was 60 Inches; therefore, set the *Gage-Point*, (viz. 52.32) upon D, to 1 upon C, then against 60 upon D, is 1.315 the Area upon C. And the Rule being thus set, against any Diameter upon D, you have the Area upon C.

The

Sect. XI. Gauging of Malt.

191

The Diameter and Depth of any Cylindrical Vessel being given in Inches, the Content in Bushels, is readily found at once. Thus: Set the *Gage-Point* upon D, to the Depth upon C; then against the Diameter upon D, is the Content upon C.

In the last Example the Diameter was 60, suppose the Depth 40 Inches, the Content will be 52.6 Bushels, and so for any other.

Having above declared what the Content of the *Winchester*-Bushel is, I shall here, for the satisfaction of the Inquisitive Reader, give an Account how the Content of the said Bushel came to be ascertained.

By the Statute of the 11 *Hen. VII. Cap. 4.* It is enacted, That Standard-Weights and Measures be made, and sent to the several Cities, Burroughs, and Market-Towns therein mentioned, which was accordingly done. But the Statute of the 12 *Hen. VII. Cap. 5.* recites, that upon more diligent Examination, the said Standard-Weights and Measures were found to be defective. And Enacts, That the Measure of a Bushel shall contain 8 Gallons of Wheat, and that every Gallon contain 8 Pounds *Troy* of Wheat, and that every Pound contain 12 Ounces *Troy* Weight, and every Ounce contain 20 *Sterlings* [now 20 Penny Weight] and every *Sterling* [or Penny Weight] be of the Weight of 32 Corns of Wheat, that grew in the middle of the Ear of Wheat. And that a Standard of a Bushel and a Gallon, after this Assize be made, and kept in the King's Treasury for ever. And new Bushels and Gallons were ordered to be made, and sent to the several Cities, Burroughs, &c. and the old ones brought and broken.

Accord-

Accordingly there remains now in the Custody of the Chamberlains of the *Exchequer* a Standard Brass Bushel, and a Standard Gallon agreeing thereto. Upon the Bushel there is this Inscription,

*Henricus Septimus Dei Gratia Rex
Angliae & Franciae.*

In *February* 1696. when a Bill was depending in Parliament, for laying a Duty upon *Malt*, I did, with the assistance of Mr. *George Tollet*, Mr. *Phil. Shales*, and Mr. *Tho. Jett*, and in the Presence of several Members of the House of Commons, make an Experiment in order to find the true Content of the said Standard-Bushel in the *Exchequer*: And the said Brass Bushel being exactly filled with common Spring-Water, and the said Water being measured out again by a Regular Parallelepipedon (whose Base was four Inches square, and the Depth 14 Inches) was found to contain 2145.6 solid Inches. The said Water was also weighed by the Standard Weights in the *Exchequer*, (and by a Beam which would turn with six Grains, with thirty Pounds in each Scale) and found to be 1131 Ounces and 14 Penny Weight *Troy*.

Now to settle the Dimensions for a Bushel to answer the said Standard, these were agreed upon as most convenient, *viz.* 18.5 Inches Diameter, and 8 Inches deep; for a Cylindrical Vessel of these Dimensions will contain 2150.42 solid Inches, which exceeding the Content of the Standard Bushel but 4.82 Inches, and there being no other convenient Dimensions (without counting to the hundredth part of an Inch) that would come so near as these: It was Enacted, in the Act for laying Duty upon *Malt*,
Tho

Sect. XI Gauging of Malta

193

That every Round Bushel, with a plain and even Bottom, being eighteen Inches and a half Diameter throughout, and eight Inches deep, shall be esteemed a legal Winchester Bushel, according to the Standard in His Majesty's Exchequer.

When this Experiment was made at the Exchequer, we also compared the Standard Troy Weights with the Standard Averdupoise Weights, and found that 15 Pound of Averdupoise, were equal to 18 Pound 2 Ounces, and 15 Penny-weight Troy; so that 140 Ounces Averdupoise, are equal to 118. 75 Ounces Troy, which in the least Terms in whole Numbers are,

Averd.

192

144

Troy

175

175

Ounces.

Pounds.

The Bushel, as now settled, contains 2150.42 solid Inches, as was shewn above, and will contain of common Spring-Water 1134.344 Ounces Troy. For,

As 2145.6, the Inches in the Brals Standard, is to 1131.7 the Troy Ounces of Water it contained; So is 2150.42, the Content of a Bushel, as now settled, to 1134.344.

Hence the following Table shews the Content, in solid Inches, of a Bushel, half Bushel, Peck, and Gallon, and likewise the weight of Water that each will contain.

K

Inches

	Inches.	Troy.			Averdupois.		
		lb.	ou.	p. w.	lb.	ou.	dr.
Bushel.	2150.42	94.	6.	6. 88	77.	12.	8. 60
Half Bush.	1075.21	47.	3.	3. 44	38.	6.	4. 30
Peck.	537.60	23.	7.	11. 72	19.	7.	2. 15
Gallon.	268.80	11.	9.	55. 66	9.	11.	9. 05

By this Table it is easie to make any of the Measures therein mentioned, or to try such as are already made, by the help of a good pair of Scales, and true weight. But where these are not to be had, the following Table shews the Diameters and Depths that will answer to the Bushel, half Bushel, Peck, and Gallon, provided they be equally wide from the Top to the Bottom.

Example. To find whether a Bushel be equal to the Standard, or not, suppose the Diameter be

Bushel

Bushel.		Half Bu.		Peck.		Gallon.	
Diam.	Depth.	Diam.	Depth.	Diam.	Depth.	Diam.	Depth.
17.5	8.94	13.7	7.30	10.7	5.98	8.3	4.97
17.6	8.84	13.8	7.20	10.8	5.87	8.4	4.85
17.7	8.74	13.9	7.09	10.9	5.76	8.5	4.74
17.8	8.64	14.0	6.98	11.0	5.66	8.6	4.63
17.9	8.55	14.1	6.89	11.1	5.56	8.7	4.52
18.0	8.45	14.2	6.79	11.2	5.46	8.8	4.42
18.1	8.36	14.3	6.69	11.3	5.36	8.9	4.32
18.2	8.27	14.4	6.60	11.4	5.27	9.0	4.23
18.3	8.18	14.5	6.51	11.5	5.18	9.1	4.13
18.4	8.09	14.6	6.42	11.6	5.09	9.2	4.04
18.5	8.00	14.7	6.34	11.7	5.00	9.3	3.96
18.6	7.91	14.8	6.25	11.8	4.91	9.4	3.87
18.7	7.83	14.9	6.17	11.9	4.83	9.5	3.79
18.8	7.75	15.0	6.08	12.0	4.75	9.6	3.71
18.9	7.67	15.1	6.00	12.1	4.68	9.7	3.64
19.0	7.58	15.2	5.93	12.2	4.60	9.8	3.56
19.1	7.50	15.3	5.85	12.3	4.52	9.9	3.49
19.2	7.42	15.4	5.77	12.4	4.45	10.0	3.42
19.3	7.35	15.5	5.70	12.5	4.38	10.1	3.36
19.4	7.27	15.6	5.63	12.6	4.31	10.2	3.29
19.5	7.20	15.7	5.56	12.7	4.24	10.3	3.23

19 Inches, and one tenth part of an Inch, find this in the first Column, and against it you have 7.5, that is, seven Inches and a half; now if the Bushel be exactly so deep, it is a legal Bushel; if the Depth be more, it may be cut to this Depth. If the Bushel be not exactly round, and the Difference of the Diameters be not above an Inch,

K 2

add

196 *Short Rules for Gauging of Malt.*

add the longest and shortest Diameters together, and take half the Sum for the Mean Diameter, and proceed as above.

Suppose the Diameter of a Bushel be 19.5 Inches, and the Depth but 7 Inches, you will find by the Table the Depth should be 7.2 Inches, from whence you may conclude this Bushel is less than the Standard; now to find how much it wants, say,

As 7.2 inches, the Depth in the Table, is to 64, the Pints in a Bushel,

So is 7 Inches, the Depth of the Bushel given, to 62.22.

That is 62 Pints, and almost a quarter of a Pint, which subtracted from 64, the Remainder is 1.77, that is, one Pint and three Quarters, and somewhat better, and so much this Bushel is too little.

Or thus, subtract 7, the depth of the Bushel given, from 7.2, the Depth found in the Table, the Difference is .2; then as $7.2 : 64 :: 2 : 1.777$, as above.

The Use of the Tables for Half Bushels, Pecks, and Gallons, are the same with respect to those Measures; and therefore Examples of these are needless.

The Solidity in Inches of the Legal Winchester Bushels, whose dimensions are as in page 193, is thus found:

As 1	:	3.14159	:	:	18.5	:	58.119415
Half of Circumference is	=						29.0597
Half Diameter of the Bushel							9.25
Product, = Area	—	—				268.8023	
Depth of the Bushel	—	—				8	
Content of the Bushel in Inches							2150.4184

In

Short Rules for Gauging of Malt. 197

In Page 43 is shew'd how to find the Area of Unity which is .785398; by which divide Unity, and the Quotient is 1.27324 and if the Content of any Square, as Bushel, Gallon, &c. be multiply'd by that, it will give the Divisors for a Circle in the same Measure. As for instance, if .2150 the Divisor for a Square Bushel be multiply'd by 1.27324, the Product is .2737.47 the Divisor for a Circular Bushel, and the Square Root of that is 52.32, the *Gage-Point* on the *Sliding-Rule* as marked with M R, signifying Malt Round, and the Square Root of .2150 is 46.37 the *Gage-Point* for the Square Bushel, and on the Line D on the *Sliding-Rule* is marked with M S, signifying Malt Square, or the *Gage-Point* for the Square Bushel of Malt. The Use of this *Gage-Point* is this, viz. in Gauging a square Cistern, Couch, or Floor, if they be not truly a Geometrical Square, then find a Mean between the Length and Breadth (as has been shewed in Page 34;) then for the Content in Malt-Bushels, say, As the *Gage-Point* on the Line D, 46.37 M S. Is to the depth, in Inches, on C, So is the Geometrical Mean between the Length and Breadth, on D

To the Content in Bushels, on the Line C.

Example. Admit a Malt-Cistern 85 inches long and 54.4 broad, Depth 36; what is the Content in Malt-Bushels?

First, the Geometrical Mean between 85 and 54.4 is 68.

Then by the Rule,

D.	C.	D.	C.
M S.	Depth.	Mean.	Content.
As 46.37 :	36 :	68 :	77.4

The like may be done for a Couch or Floor-Gage,

A Cistern, Couch, or Floor-Gage may be more speedily and expeditiously cast up by a Line which

198 *Short Rules for Gauging of Malt.*

is now put on the *Sliding-Rule* marked with MD signifying Malt-Depth ; this Line is no other than a Line of Numbers in a Reverse Order, and its use is so exceeding great for Casting up Malt-Gauges, that to omit it, where to be guilty of one of the greatest Crimes that could be Committed in the whole Book.

On this Line always find the Depth of the Cistern, Couch or Floor ; Then say, As the Depth upon the Line MD, is to the Length or Breadth upon B, So is the Length or Breadth upon A, To the Content in Malt-Bushels upon B. *Example.* Admit a Floor whose Length is 160, Breadth 132, and Depth 8.1 Inches ; what is the Content in Malt-Bushels ?

Operation by this Sliding-Rule.

Set the Depth 8.1 on M D, to the Length 160 on the Line B ; then against the Breadth 132 on the Line A you have 79.5 upon the Line B, the Content in Malt-Bushels.

Thus,

	MD	B	A	B
As	8.1	: 160	: : 132	: 79.5
	Depth.	Length.	Breadth.	Content.

Or thus,

	MD	B	A	B
As	8.1	: 130	: : 160	: 79.5
	Depth.	Breadth.	Length.	Content.

More Examples in a thing so plain were needless.

SECT.

S E C T. XII.

The USE of the
R U L E
In Questions of
Simple Interest.

Problem. I.

To find the Interest of any Sum of Money for any time proposed (in Years, Months, or Days) at any Rate of Interest.

1. For Years.

exam. **W**hat is the Interest of 350*l.* for five Years, at 6 per Cent. per Ann.?

1. Set 1 upon A, to .06 the Rate upon B, then against 5, the Time upon A, is .3, the Interest of 1*l.* in five Years upon B.

K 4

2. Set

2. Set 1 upon A, to 3 upon B, then against 350, the Sum given, upon A, is 105 upon B, that is 105 £ the Interest of 350 £ . in five Years, which was required.

2. For Months.

Exam. **W**hat is the Interest of 350 £ . for four Months, at 6 per Cent. per Ann. ?

Set 12 upon A, to .06 the Rate upon B, then against 4, the Months given upon A, is .02, the Interest of 1 £ . in four Months upon B.

2. Set 1 upon A, to .02 upon B, then against 350 the Sum given upon A, is 7 upon B; that is, 7 £ . the Interest sought,

3. For Days.

Exam. **W**hat is the Interest of 350 £ . for seventy three day, at 6 per Cent per Ann. ?

1. Set 365 upon A, to .06 the Rate upon B, then against 73 the Days given upon A, is .012 the Interest of 1 £ . 73, days upon B.

2. Set 1 upon A, to .012 upon B, then against 350 the Sum given upon A, is 4.2 upon B; that is, 4 £ . 4 s . the Interest sought.

There are several other useful Questions relating to Simple Interest, some of which shall be resolved in the following Problems. In order to which, put

Sect. XII. Of Simple Interest.

201

u = A Principal or Sum forborn.

t = The Time of Forbearance in Years, or part of a Year.

g = The Gain or Rate of Interest.

a = The Amount of the said Principal, in the said Time, and at the said Rate of Interest.

Then if any three of these Particulars be given, the fourth may be found.

Problem II.

Given u , g and t : To find a .

$$1 : g t + 1 :: u : a.$$

Exam. **W**hat will 250 l . amount unto in 6 Years, at 6 per Cent. per Annum?

$u = 250$, $g = .06$, $t = 6$, and

$g t + 1 = 1.36$, Therefore,

Set 1 upon A, to 1.36 upon B, then against 250 upon A, is 340 upon B, the Amount sought.

Problem III.

Given a , g and t : To find u .

$$g t + 1 :: 1 : a : u.$$

Exam. **W**hat is 340 l . due 6 Years hence worth in ready Money, at 6 per Cent. per Ann.

K 5

$a =$

$$a = 350, g = .06, t = 6$$

and $g \cdot t = 1.36$ Therefore,

Set 136 upon A, to 1 upon B, then against 340 upon A, is 250 upon B, the Present Worth sought.

Problem VI.

Given a, u , and g : To find t .

$$u g : 1 :: a - u : t.$$

Examp. **I**N how many Years will 250 *l.* amount unto 340 *l.* at 6 per Cent. per Ann. ?

$$a = 340, u = 250, g = .06.$$

$u g = 15, \text{ and } a - u = 90.$

Therefore, Set 15 upon A, to 1 upon B, then against 90 upon A, is 6 upon B, that is, 6 Years the Time sought.

Given a , u and t . To find g .

At: I: : a - - - - - : g.

u=340, a=350, t=6
u t=1500 and a -- u=120

The *Interest of Money for Days*, requiring more Exactness than can be expected from any Instrumental Operation, here is added a short Table, which readily shews the *Interest of any Sum for any Time*, and at any Rate of Interest, to the hundredth part of a Parthing and is also very useful for other purposes.

[illegible]

A TABLE of *Simple Interest* for any Sum, Time,
and Rate.

l. s. d. f.			s. d. f.		
1000000	2739	15 6 0.99	90	4 11 0.71	
900000	2465	15 0 3.29	80	4 4 2.41	
800000	2191	15 7 1.59	70	3 10 0.13	
700000	1917	16 1 3.89	60	3 3 1.81	
600000	1643	16 8 2.19	50	2 8 3.51	
500000	1369	17 3 0.49	40	2 2 8.21	
400000	1095	17 9 2.79	30	1 7 2.98	
300000	821	18 4 1.09	20	1 1 0.60	
200000	547	18 10 3.40	10	0 6 2.30	
100000	273	19 5 1.70			
90000	246	11 6 0.33	9	0 5 3.67	
80000	219	3 6 0.96	8	0 5 1.04	
70000	191	15 7 1.59	7	0 4 2.41	
60000	164	7 8 0.22	6	0 3 3.78	
50000	136	19 8 2.85	5	0 3 1.15	
40000	109	11 9 1.48	4	0 2 2.52	
30000	82	3 10 0.11	3	0 1 3.89	
20000	54	15 10 2.74	2	0 1 1.26	
10000	27	7 11 1.87	1	0 0 2.63	
9000	24	13 1 3.23	.90	0 0 2.37	
8000	21	18 4 1.18	.80	0 0 5.10	
7000	19	3 6 2.96	.70	0 0 1.84	
6000	16	8 9 0.82	.60	0 0 1.58	
5000	13	13 11 3.68	.50	0 0 1.32	
4000	10	19 3 0.55	.40	0 0 1.05	
3000	8	4 4 2.41	.30	0 0 0.79	
2000	5	9 7 0.27	.20	0 0 0.53	
1000	2	14 9 2.14	.10	0 0 0.26	
900	2	9 3 3.12	.09	0 0 0.24	
800	2	3 10 0.11	.08	0 0 0.21	
700	1	18 4 1.10	.07	0 0 0.18	
600	1	12 10 2.08	.06	0 0 0.16	
500	1	7 4 3.07	.05	0 0 0.13	
400	1	1 11 0.05	.04	0 0 0.11	
300	0	16 5 1.04	.03	0 0 0.08	
200	0	10 11 2.03	.02	0 0 0.05	
100	0	5 5 3.01	.01	0 0 0.02	

The Use of the Table.

I. To find the *Interest* of any *Sum*, for any number of *Days*, at any *Rate* of *Interest*.

Multiply the *Sum* given by the *Rate*, and that *Product* by the number of *Days*, then set down the *Figures* of the last *Product*, with *Cyphers* towards the right *Hand*, as in the *Example* following, the *Sums* answering to these *Numbers* taken out of the *Table*, added together give the *Interest* sought.

Example. What is the *Interest* of 6525 l. for 146 *Days* at 1 per Cent. per *Ann*?

Principal ————— 6525
Rate ————— .08

1. Product ————— 521.00
Number of Days — 146

31320

20880

521

2. Product ————— 76212

70000 ————— 191 15 7 1 59

6000 ————— 16 8 9 0 82

200 ————— 0 10 11 2 03

10 ————— 0 0 6 2 30

2 ————— 0 0 1 1 26

Answer ————— 208 16 0 0 00

2. Example, What is the Interest of 825 l. 10 s. in 146 Days, at 8 per Cent. per Ann.

Principal ——— 825. 5
Rate ——— .08

1. Product ——— 66.040
Num. of Days ——— 146

39624
26416
6604

2. Product ——— 9641.84

	l.	s.	d.	f.
9000 ———	24	13	1	3. 23
600 ———	1	12	10	2. 08
40 ———	0	2	2	1. 21
1 ———	0	0	0	2. 63
.80 ———	0	0	0	2. 10
.04 ———	0	0	0	0. 11

Answer ——— 26 8 3 3 36

3. Example. What is the Interest of 825.5 l. at the same Rate, for the Remainder of the Year, viz. 219 Days?

1. Product ——— 66.04
Number of Days ——— 219

59436
6604
13208

2. Product ——— 14462.76

Sect. XII. Of Simple Interest

8007

10000	—	—	27	7	11	11.37
4000	—	—	10	19	2	0.55
400	—	—	1	11	11	0.05
60	—	—	0	3	3	1.81
10	—	—	0	0	1	1.25
70	—	—	0	0	0	0.84
80	—	—	0	0	0	0.16

The Interest of 825.5 l.
for 219 Days, at 8 per Cent. is } 39 12 5 3 04

The Interest of 825.5 l.
for 146 Days, at the
same Rate, was found
to be } 26 8 3 3 36

The Sum is the exact
Interest of 825.5 l. for
365 Days, or one Year
at 8 per Cent. per Ann.
vix. } 66 20 9 2 40

II. Any Annual Salary, or Pension, being given, to
find what it amounts to by the Day.

Against any Annual Salary in the first or third
Col. of the Table, you have the Amount for one
Day, in the second or fourth Col. respectively.
Thus,

		l	s	d	f	
5000	pounds	13	13	11	2.68	} per Day.
600	per An.	1	12	10	2.08	
70	is	0	3	10	0.11	

And

And so for any other Sum, found in the first or third Column of the Table.

And so for any number of Days; multiply the Annual Salary by the number of Days given, and place the Figures of the Product, with Cyphers towards the right Hand, as in the Example following, the Sums answering to these Numbers taken out of the Table, and added together, give the Answer.

Example. At a Salary of 525 l. per Ann. what is due for 73 Days?

Salary ——— 525
Days ——— 73

1575

3675

38325

30000 ——— 82

8000 ——— 21

300 ——— 0

50 ——— 0

20 ——— 0

Answer 105

3 10 0. 11

4 1. 10

5 1. 04

0 1. 15

1 1 0. 60

0 0 0. 00



Of Annuities at Simple Interest.

Problem VI.

To find the Amount of any Annuity for any Time given, and at any Rate of Simple Interest.

A General Rule.

1. Find the Interest of the Annuity for half the number of Years, half Years or Quarters, that the Annuity is to continue.

2. Multiply the Interest thus found by the number of Years, half Years, or Quarters less by 1, and to this Product add the Sum of the Annual, half yearly, or quarterly Payments, the Sum will be the true Amount of the Annuity.

Example: What is the Amount of an Annuity of 100 £ in 8 Years, at 6 per Cent?

The Interest of 100 £ for half the time, viz. 4 Years, is 24 £ . this multiplied by the number of Years less by 1, (viz. 7.) the Product will be 168, to which add the Sum of the Annual Payments, viz. 800, the Sum will be 968 £ . the Amount sought.

Pro-

Problem VII.

To find the present worth of any Annuity for any given Time, at any Rate, accompting Simple Interest.

The Rule, or Proportion is

As the Amount of 1 l. in any Time is to 1 l.
So is the Amount of any Annuity, to the present worth.

Therefore, Divide the Amount of the Annuity by the Amount of one Pound in the Time given, the Quotient is the present Worth.

Example. What is the present Worth of an Annuity of 100 l. to continue 8 Years, at 6 per Cent.

The Amount of 100 l. Annuity for 8 Years, by the last Problem, was found to be 968 l. and the Amount of one Pound in 8 Years, will be found (by the II. Problem before-going) to be 1.49.

Now if 968 (the Amount of the Annuity) be divided by 1.49, the Quotient is 654.054, and so much present Money is equal in Value to an Annuity of 100 l. for 8 years; for if this 654.054 be put out at Interest it will in 8 Years amount to 968 l. at 6 per Cent. per Ann. Several Writers have treated of the Calculation of Annuities at Simple Interest; but in as much as all present Worths either of Principal Sums, or Annuities due at any Time to come, ought to be consider'd in a Geometrical Proportion, according

according to the several Rates of Compound Interest, all Computations of Annuities at Simple Interest, are to be laid aside as useless.

Of Compound Interest.

IN Compound Interest, the respective Amounts for each respective Year are so many Geometrical proportional Numbers. Thus, if 100 l. be put out to Interest at 6 per Cent. per Annum, there will be due at the end of the first Year 106 l. which 106 is now made a Principal, and at the end of the second Year will amount to 112.36; and this being made a Principal, will at the end of the third Year be increased to be 119.1016; now these four Numbers, viz. 100, 106, 112.36, and 119.1016, are four Geometrical proportional Numbers: For,

As 100 to 106, (for the first Year);
So is 106 to 112.36 (for the second Year.)

Again, As 106 to 112.36;
So is 112.36 to 119.1016, for the third Year, &c.

Pro-

Problem. VIII.

Any Sum of Money being propounded, to find the Amount thereof in any time, at any Rate of Compound Interest.

Example: What is the Amount of 100 l. being forborn 5 Years, at 6 per Cent. per Ann. Compound Interest?

1. **S**ET 100 upon B, to 106 (the first Years Amount upon A; then against 106 upon B, you have 112.36 upon A, the Amount in two years. *In like manner,*

2. Against 112.36 upon B, is 119.1016 upon A, the Amount in three Years. *Again,*

3. Against 119.1016 upon B, is 126.247 upon A, the Amount in four Years.

Lastly, Against 126.247 upon B, you have 133.822 upon A, which is the Amount in 5 Years, and the Answer to the Question.

Problem IX.

A Sum of Money being due at any time to come, to find what it is worth in ready Money, at any Rate of Interest,

Example: What is the present worth of 133.822 l. due at the end of 5 Years to come, at 6 per Cent. per Annum Compound Interest?

THIS is but the Inverse of the last Problem: Therefore if you set the Rule as is there directed, you may read the Proportion back again: For the respective Amounts found upon A, in that Example, will point you to the respective present Worth at the end of each respective Year.

Thus, Set 106 upon A, to 100 upon B; then against 133.822 upon A, is 126.247 upon B: The present Worth of 100 l. at the end of the first Year.

Also, Against 126.247 upon A, you have 119.1016 upon B, the present Worth at the end of the second Year.

Again, Seek 119.1016 upon A, against it you have 112.36 upon B, the present Worth at the end of the third Year.

In like manner, Against 112.36 upon A, is 106 upon B, the present Worth at the end of the fourth Year.

Lastly, Against 106 upon A, you have 100 upon B, which is the present Worth of 133.822 l. at the end of five Years, which was required.

Pro-

Problem X.

A yearly Rent or Annuity being forborn a certain number of Years, to find what the Arrearages thereof will amount unto at any Rate propounded.

1. **F**ind a Sum (or Principal) whose Interest is equal to the Annuity or yearly Rent.
2. Find (by *Probl. VIII.*) the Amount of that Sum in the time given : This done, subtract the Principal first found from the Amount, the Remainder will be the Answer.

Example : A Rent or Annuity of 9 l. per Annum being forborn 22 Years, what will the Arrearages thereof amount unto, accounting Interest upon Interest, at 6 per Cent ?

To find a Principal whose Interest is 9 l. say,

As 6 to 100 ;

So is 9 to 150, the Principal sought.

The Amount of 150 l. in 22 Years, will be found by Problem VIII, to be 301.828, from which subtract (the Principal first found, viz.) 150, the Remainder will be 151.828, the Sum of the Arrearages sought.

Pro-

Problem XI.

A yearly Rent or Annuity being propounded, to find what it is worth in ready Money.

First, find (by the last Problem) what the Arrearages thereof will amount unto at the end of the Term propounded: Then (by *Probl. IX.*) find the present Worth of those Arrearages: The present Worth thus found is the value of the Annuity or Yearly Rent sought.

Example: Suppose a Rent of 9 l. per Ann. to continue 12 Years, What Sum of Money may be given for this Rent, allowing the Buyer 6 per Cent. Compound Interest?

By the last Problem the Arrearages of a Rent of 9 l. per Ann. being forborn 12 Years, were found to be 151.818 l.

And the present Worth of 151.818, due at the end of 12 Years to come, will be found (by *Probl. IX.* to be 77.443, and so much may be given for a Rent of 9 l. per Ann. to continue 12 Years.

But suppose the Term of the Rent or Annuity begins not till after the Expiration of certain Years to come, in such case, find what the Arrearages forborn for all that time are worth in ready Money, for that is the Answer.

For Instance.

Suppose in the last Example, the Annuity of 9 *l.* for 12 Years, were not to begin till after the Expiration of 6 Years: I here seek what the Arrearages, (*viz.* 151.828) being forborn 18 Years are worth in ready Money, and by *Problem IX.* I find it will be 53.185 *l.* which is the Answer: So a Yearly Rent or Annuity of 9 *l.* per Annum, to begin 6 Years hence, and continue 12 Years, is worth but 53.185, in ready Money.

Problem XII.

A Sum of Money being propounded, to find what Annuity (to continue any any Number of Years, and at any Rate proposed) that Sum will buy.

TAKE any Annuity at pleasure, and find the Value thereof in ready Money: This done, the Proportion is,

*As the present Worth found, is to the Annuity taken;
So is the Sum given, to the Annuity sought.*

Example: *What Annuity to continue 12 Years will 300 *l.* Purchase, allowing the Buyer 6 per Cent. Compound Interest?*

By *Problem XI.* I find that 75.443 will purchase 9 *l.* per Annum, to continue 12 Years.

Therefore, Set 75.443 upon B, to 9 upon A; then against 300 upon B, is 35.776 upon A, that is 35 *l.* 15 *s.* 6*d.* and so much Annuity to continue 12 Years is worth 300 *l.*

Pro-

Sect. XII. Of Compound Interest.

217

Problem. XIII.

There is a Freehold Estate of the yearly Value of 78 l. What Sum of Money is the Estate worth, allowing the Buyer 6 per Cent. per Ann. Compound Interest for his Money?

The Rule or Proportion is,

As the Annual, half Yearly, or Quarterly Interest of 1 l. is to 1 l.;

So is the Annual, half Yearly, or Quarterly Rent to the Sum sought.

The Interest of 1 l. at 6 per Cent. Compound Interest,

For 1 { Year } is { .06 }
 { Half Year } { .01956 }
 { Quarter } { .01467 }

Now to Answer the Question.

Set .06 upon B, to 1 upon A; then against 78 upon B, is 1300 upon A, which is the Value of the Estate propounded; and for the same reason I find without moving the Rule, that a Freehold Estate.

of { 90 } Pounds { 1500 }
 { 120 } per Ann. { 2000 }
 { 150 } is worth { 2500 } Pounds.

L

But

But suppose the Rent of the former Estate, (*viz.* 78 *l.*) were payable Quarterly, (*viz.* 19.5 *l.* per Quarter :) What is it then worth, allowing the Purchaser 6 per Cent. as before ?

Set .01467. (the Interest of 1 *l.* for a Quarter of a Year) upon A, to 1 upon B ; then against 19.4 (the Quarter's Rent) upon A, you have 1329.244 upon B, that is 1329 *l.* 4 *s.* 10 $\frac{1}{2}$ *d.* the Answer, which is 29 *l.* 4 *s.* 10 $\frac{1}{2}$ *d.* more than the former ; that being accounted by Yearly, this by Quarterly Payments.

Problem XIV.

A Sum of Money propounded, to find the Annual Rent of a Free-hold Estate that such a Sum will purchase, at a given Rate of Compound Interest.

This is the Inverse of the last, and therefore the Proportion is :

As 1 to the Annual Interest ;

So is the Sum propounded to the Annual Rent.

Examples. Suppose I would lay out 3650 *l.* upon a Free-hold Estate, so as to have 6 per Cent. Comp. Interest for my Money, What must the Annual Rent be ?

Set 1 upon A, to .06 upon B ; then against 3650 upon A, you have 219 upon B : The Answer is 219 *l.* per Annum.

Pro-

Problem. XV.

The Annual Rent of a Freehold Estate being known, and also the Sum of Money which was paid for the same, to find what Rate of Compound Interest the Purchaser shall have for his Money.

The Proportion is,

A S the Sum given is to 1;
So is the Annual Rent to the Rate of Interest.

Example: There is 1300 L. given for a Free-hold Estate, whose Annual Rent is 78 l. What Rate of Interest has the Purchaser for his Money?

Set 1300 upon A, to 1 upon B; then against 78 upon A, is .06 upon B: So the Purchaser shall have 6 per Cent, for his Money.

If it be demanded, How many Years Purchase a Freehold Estate is worth at any Rate of Compound Interest?

The Proportion is,

As the Rate of Interest is to 1;
So is 1 to the Answer.

Example, at 5 per Cent.

Set .05 upon B, to 1 upon A; then against 1 upon B, is 20 upon A; so at 5 per Cent. it is worth 20 Years Purchase.

At 6 per Cent. it is worth but 16.66 Years Purchase. For,

As .06 upon B, to 1 upon A;
So is 1 upon B, to 16.66 upon A; that is, 16 Years
8 Months, and somewhat more.

The Converse of this is, to find by the Number of Years Purchase what Rate of Interest the Buyer hath for his Money.

Say, As the Number of Years Purchase is to 1;
So is 1 to the Rate of Interest.

Example: A free-hold Estate is bought at 20 Years Purchase; What Rate of Interest hath the Purchaser for his Money?

Set 20 upon A, to 1 upon B, then against 1 upon A, is .05 upon B; that is, 5 per Cent. per Ann.

SECT.

S E C T. XIII.

The Rule of Fellowship.

Problem. I.

Five Persons CDEF and G, make a Bank or Stock of Money, in all 360 l. with which they Trade and gain 40 l. what part of this Gain must each Person receive to balance his Money put in?

THE Instrument will resolve this and such like Questions by Inspection.

The Proportion is,

As the whole Stock;

is to the whole Gain;

So is each Man's particular Stock,
to his particular Gain.

Therefore, Set 360 the whole Stock upon A, to 40 the whole Gain upon B, then against each Person's Stock upon A, you have his Gain upon B. Thus against 108 upon A, you have 12 upon B. So C who put 108 l. shall have 12 l. of the Gain, and so of the rest.

L 3

Stock

	Stock.	Gain.
C	108	12
D	81	9
E	63	7
F	72	8
G	36	4
	360	40

By which it appears, that this Question (which by the Pen would require five Operations) is resolved by the Rule at one.

A Question of this Rule with Time.

Problem II.

Two Merchants join together, H put in 60 l. for 6 Months, F put in 80 l. for 8 Months; and they gained 50 l; How much must each Merchant have of the Profit ?

The Proportion will be,

(1) As 1 is to 6 (Months);

So is 60 l. to 360.

(2) As 1 is to 8 (Months);

So is 80 l. to 640.

Set 1 upon A, to 6 upon B; then against 60 upon A, is 360 upon B.

Again,

Se&t. XIII. The Rule of Fellowship. 223

Again, Set 1 upon A, to 8 upon B; then against 80 upon A, is 640 upon B, the Sum of these is 1000.

Now for each Merchant's Share.

1. Set 1000 upon A, to 50 upon B; then against 360 upon A, is 18 upon B, that is 18 l. and so much is H's Share.
2. Set 1000 upon A, to 50 upon B; then against 640 upon A, is 32 upon B, which is the Share belonging to F.

So that $\begin{Bmatrix} H \\ F \end{Bmatrix}$ must have $\begin{Bmatrix} 18 \text{ l.} \\ 32 \text{ l.} \end{Bmatrix}$

The whole Gain 50 l.

 S E C T. XIV.

The Use of the RULE

 I N
 Military AFFAIRS.

Problem I.

Any number of Men being proposed, to place them in Battalia, so that there may be as many in Rank as in File, and supposing them to be im-battelled at Order (which is 3 Foot distance in Rank and File) to find how many square Foot of Ground they will take up?

Example. **L**ET the Number of Men proposed be 3600. Set the Lines C and D even at the end. Now forasmuch as the Number given doth consist of an even Number of Figures, seek the Number, viz. 3600 in the second Radius on the Line C, and against it on the Line D, you will

Sect. XIV. Of Military Affairs.

225

will find, 60 for many Men must be in Rank, and as many in File.

Now to find how many square Foot of Ground they will occupy :

Set 1 upon A to 3 upon B, then against 60 upon A is 180 upon B.

Then set the Lines C and D even, and find this Number, viz. 180 on the first Radius on the Line D, and against it you have 32400 upon C, and so many square Foot of Ground will these Soldiers occupy, being at their Order of three Foot.

Problem II.

Any Number of Men being proposed, to place them in Battalia, in such order that there may be twice as many in Rank as in File, and that they may stand at close Order, which is $1\frac{1}{2}$ Foot:

Example. **L**ET the Number given be 1800, set the Lines D and C even, and count half the Number given, viz. 900, upon the Line C, and against it you have 30 upon D, which is the Depth of the File, and then there must be 60 in Rank, which is twice 30.

Now for the Ground these will occupy being at close Order.

Set 1 upon A to 1.5 upon B, then against 30 upon A is 45 the side of the Ground, ——— and without moving the Lines, seek 60 upon the Line

L 5.

A,

A, and against it you have 90 upon B, which is the Length of the Ground.

Now for the Area.

Set 1 upon A, to 45 upon B; then against 90 upon A, is 4050, and so many square Foot of Ground will these 1800 Men occupy at Close Order.

Problem III.

Any Number of Men being proposed to be put in Battalia, and a certain Number named to be either in Rank or File to find the other Number.

Example. **L**ET it be required to place 1200 Men in Battalia, so that there shall be 15 in File, how many must there be in Rank; or how many Files must there be?

This is resolv'd by the Lines A and B. And the Proportion is,

As 15 the Depth in File, is to 1;

So is 1200, the Number proposed, to 80 the Number of Men in Rank.

Pro-

Problem IV.

Any Number of Soldiers being given, together with their distance in Rank and File, to order them into a Square Battalia of Ground.

Example. **L**ET the Number of Soldiers given be 5400, their Distance in File 6 Foot, and in Rank 4 Foot.

Set 6 upon the Line A, to 4 upon the Line B, and then against 5400 upon A, is 3600 upon B.

Then set the Lines D and C even, and seek 3600 in the second Radius upon C, against it you have 60 upon D, the number of Men to be placed in File.

Now for the Number of Men to be placed in Rank, say,

As 60 is to 1 on the Lines A and B.

So is 5400 to 90.

So there shall be 60 in File, and 90 in Rank.

Now if you multiply 60, the Number of Men in File, by 6, their Distance, the Product will be 360, the side of your Ground.

In like manner, if you multiply 90, the Number of Men in Rank, by 4, their Distance, the Product will be 360, as above; which shews, your Ground is exactly square, and is therefore a Proof of the Work.

Problem V.

If 1000 Soldiers may be Lodged or Quartered in a Square Piece of Ground, whose side is 400 Foot; how many Foot long must the side of a Square be, that the Ground included may lodge 4000?

The Proportion will be,

(1) As Side is to 400 (the Side of the Square which will Lodge 1000 Soldiers)
So is 4000, to 160000.

(2) As 1000, is to 4000;
So is 160000 to 640000.

Now (setting the Lines D and C even) find this 640000 in the second Radius on the Line C, and against it you have 800 upon D, and so much must the Side of a Square be; to Lodge 4000 Soldiers with the same convenience that a 1000 were Lodged in a Square whose Side was 400 Foot.

SECT.

S E C T. XV.

Of Triplicate Proportion.

Triplicate Proportion, is such a Proportion as is between Lines and Solids, or between Solids and Lines, of which I shall give some Examples in the following Problems.

Problem I.

If a Bullet whose Diameter is 4 Inches, weighs 9 Pounds, what shall another Bullet of the same Metal weigh, whose Diameter is 8 Inches?

SET 4 (the Diameter of the Bullet proposed) upon the single Line, to 9 the weight on the Line E, then against 8 upon the single Line is 72, which is the weight of the Bullet whose Diameter is 8 Inches.

And as the Lines now stand, you may find the weight of all Bullets of the same Metal; for against the Diameter upon the single Line you have the weight upon the Line E.

Thus

Thus, if the Diameter of a Bullet

$$\text{be } \left\{ \begin{array}{l} 3 \\ 5 \\ 6 \\ 7 \end{array} \right\} \text{ Inches, the } \left\{ \begin{array}{l} 3.79 \\ 17.57 \\ 30.37 \\ 48.23 \end{array} \right\} \text{ Pounds.}$$

weight will be

Problem II.

Having the Weights of two Bullets of the same Metal, and the Diameter of one of them, to find the Diameter of the other.

THis is but the Converse of the last, and therefore without moving the Lines, find the Weight on the Line E, and against it you have the Diameter on the single Line.

Thus, if a Bullet

$$\text{weighs } \left\{ \begin{array}{l} 20 \\ 30 \\ 40 \end{array} \right\} \text{ Pounds, the } \left\{ \begin{array}{l} 5.23 \\ 5.97 \\ 6.67 \end{array} \right\} \text{ Inches.}$$

Diameter will be

Problem

Problem III.

If a Brass Gun, whose Diameter is 8 Inches, weighs 640 Pounds what shall another Gun weigh (of the same Shape and Metal) whose Diameter is 6 Inches?

SET 8 upon the single Line, to 640 on the Line E, then against 6 upon the single Line, is 170 on the Line E, which is the Weight of the Piece whose Diameter is 6 Inches.

Problem IV.

There is a Conical Tun which contains 620 Gallons, the Diameter at the Bottom is 80 Inches, the Diameter at the Top 71.2 Inches, and the Depth 30 Inches; it is desired to make another Tun in the same Shape with the former, that may contain 800 Gallons. To find the Diameter of this Tun at the Bottom and Top, and also the Depth.

1. For the Diameter at the Bottom.

SET 620 (the Content of the first Tun) upon the Line E, to 8 (the Bottom-Diameter) on the single Line: Then against 800 (the Content of the second Tun) upon E, is 87, which is the Diameter sought.

2. For

2. For the Diameter at the Top.

Set 620 upon E to 71.2 upon the single Line, then against 800 upon E is 77.4 the Diameter sought.

3. For the Depth.

Set 620 upon E to 30 upon the single Line, then against 800 upon E is 32.63 the Depth required.

So the Diameters and Depth of this Tun are as followeth.

Bottom	} Diameter	87	} Inches.
Top---		77.4	
Depth		32.63	

But the first of these being found as above directed, the other may be discover'd at one Operation by the Lines A and B. Thus:

Set 80 upon A to 87 upon B, then against 71.2 upon A is 77.4 upon B, and against 30 upon A is 32.63 upon B, which also shews, that the Diameters and Depth of this Tun, are in porportion to the Diameters and Depth of the Tun first proposed. And the like may be done for any Cask: of which I need not give any Examples.

Problem V.

There is a Ship of 60 Tun Burthen, which measures 44 Foot by the Keel, and it is required to make another Ship of 360 Tun Burthen, of the same Shape with the first. How many Foot long must the Keel of the second Ship be?

SET 60 upon E, to 44 upon the single Line, then against 360 upon E, is 80 on the single Line, which is the Length of the Keel required.

Now suppose in the first Ship the middle Beam be 17.6 Foot, and the Depth of the Hold 7.76 Foot.

To find these Dimensions of the second Ship,

Set 44 upon A, to 80 upon B, then against 7.76 upon A, is 32 upon B, which is the Length of the Beam. And without moving the Lines, seek 7.76 upon A, and against it you have 14.1 upon B, which is the Depth of the Hold. So the Dimensions and Burthen of the two Ships are as followeth,

	1 Ship	2 Ship	
Length of the { Keel }	44	80	} Foot.
{ Beam }	17.6	32	
Depth of the Hold	7.76	14.1	} Tun.
Burthen	60	360	

To

234 *Of Triplicate Proportion. Sect. XV.*

To find the Burthen of any Ship, whose Length, Breadth, and Depth is given in Feet, the Common Rule is,

Multiply the Length of the Keel by the Length of the Midship Beam, then Multiply the Product by the Depth of the Hold, then divide this last Product by 100, the Quotient gives you the Tuns. But when there is no Allowance for Guns, divide the last Product by .95.

Thus, in the second Ship abovementioned, multiply 80, the Length, by 32 the Breadth, the Product is 2560; this multiplied by 14. 1, the Depth, gives 36096; and this divided by 100, is 360.96, that is, 360 Tons, and 96 parts of a Ton, which is the Burthen of this Ship, allowing for Guns. But if there be no Allowance made for Guns, divide the last Product, viz. 360.96, by 95, in this case her Burthen will be found to be 379.95 that is, 380 Ton very near.

SECT.

S E C T. XVI.

PROPORTIONS IN A CIRCLE.

IT was shewn in Page 42, that if the *Diameter* of a Circle be 1, the *Circumference* will be 3.141592, and the *Area* .785398, the Square-Root of .785398 is .886227, and this is the Side of a Square, equal to the Area of a Circle, whose Diameter is 1. And the Square-Root of .5, viz. .707107 is the Side of the greatest Square that can be inscribed in a Circle, whose Diameter is 1: So that if the Diameter of a Circle be 1,

$$\text{the } \left\{ \begin{array}{l} \text{Circumference} \\ \text{Square equal} \\ \text{Squ. inscribed} \\ \text{Area} \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 3.141592 \\ .886227 \\ .707107 \\ .785398 \end{array} \right\}$$

Note,

236 Proportions in a Circle. Sect. XVI.

Note further, that if the Circumference of a Circle be 1,

$$\text{The } \left\{ \begin{array}{l} \text{Diameter} \\ \text{Sq. equal} \\ \text{Sq. Inscribed} \\ \text{Area} \end{array} \right\} \text{ will be } \left\{ \begin{array}{l} .318310 \\ .282095 \\ .225079 \\ .079577 \end{array} \right\}$$

If the Area of a Circle be 1,

$$\text{The } \left| \begin{array}{l} \text{Diameter} \\ \text{Circumference} \end{array} \right| \text{ is } \left| \begin{array}{l} 1.128379, \\ 3.54491, \end{array} \right|$$

If the Circumference of a Circle be 4,

$$\text{The } \left| \begin{array}{l} \text{Diameter} \\ \text{Area} \end{array} \right| \text{ is } 1.27324,$$

If the Diameter of a Circle be 4,

$$\text{The } \left| \begin{array}{l} \text{Circumference} \\ \text{Area} \end{array} \right| \text{ is } 12.56637,$$

In all these Cases the Area is Square Inches, Foot, Yards, &c. the other Terms being Lines, do refer to Lineal, Inches, Foot, Yards, &c. respectively. These Things premised, I shall here add some useful Questions, which may be resolved either by the Pen or Instrument,

SECT. XVI. Proportions in a Circle. 237

1. The Diameter of a Circle given, to find the Circumference.

As 1 is to 3.141593, Or,

As 3.1831 is to 1;

So is the Diameter to the Circumference.

2. The Circumference given, to find the Diameter,

As 1 is to .31831, Or,

As 3.141593 is to 1;

So is the Circumference to the Diameter.

3. The Diameter given, to find the Side of a Square, equal to the Circle.

As 1 is to .886227, Or,

As 1.128379 is to 1;

So is the Diameter to the Side of a Square equal.

4. The Side of a Square given, to find the Diameter of a Circle whose Area is equal to the Square.

As 1 is to 1.128379, Or,

As .886227 is to 1;

So is the side of the Square to the Diameter sought.

5. The Circumference given, to find the side of a Square equal to the Circle.

As 1 is to .282095, Or,

As 3.54491 is to 1;

So is 1 the Circumference to the side of the Square required.

6. The

6. The Side of a Square given, to find the Circumference of a Circle equal to the Square.

As 1 is to 3.54491, Or,

As .282095, is to 1 ;

So is the side of the Square to the Circumference sought.

7. The Diameter of a Circle given, to find the side of the greatest Square inscribed.

As 1 is to .707107, Or,

As 1.41421 is to 1 ;

So is the Diameter to the Side required.

8. The side of a Square given, to find the Diameter of a Circle that shall circumscribe the Square.

As 1 is to 1.41421, Or,

As .707107 is to 1 ;

So is the Side of the Square to the Diameter sought.

9. The Circumference of a Circle given, to find the Side of the Square inscribed.

As 1 is to .225079, Or,

As 4.4429 is to 1 ;

So is the Circumference to the Side required.

10. The

Sect. XVI. Proportions in a Circle. 239

10. *The Side of a Square given, to find the Circumference of a Circle that shall circumscribe the Square.*

As 1 is to 4.4429, Or,

As .225079 is to 1;

So is the Side of the Square to the Circumference sought.

11. *The Diameter of a Circle given to find the Area.*

As 1 is to .785398, Or,

As 1.27324 is to 1;

So is the Square of the Diameter to the Area required.

12. *The Area of a Circle given, to find the Diameter:*

As 1 is to 1.27324, Or,

As .785398 is to 1;

So is the Area to the Square of the Diameter, whose Square-Root is the Diameter sought.

13. *The Circumference of a Circle given, to find the Area.*

As 1 is to .079577, Or,

As 12.56636 is to 1;

So is the Square of the Circumference to the Area sought.

14. *The*

14. The Area of a Circle given, to find the Circumference.

As 1 is to 12.56637, Or,

As .079577 is to 1,

So is the Area to the Square of the Circumference, whose Square-Root is the Circumference required.

Such of these Proportions as relate to finding the Area of a Circle by the Diameter or Circumference, being of most general use, and requiring, in some cases, more exactness than can be attained to by the Numbers here given. I have here added two Tables for that purpose, which were Calculated to sixteen Places, and are here Printed exactly true to ten Places in the Fractions, only the last Figure is made one more, where the eleventh Figure was above five. See Figure 24.

A TABLE shewing the Area and Circumference of any Circle in Inches, Feet, Yards, &c. to any Diameter in Inches, Feet, Yards, &c. from 1 to 12, or from 10 to 120.

Dia.	Area.	Circumference.
1.0	.7853981634	3.1415926536
1.1	.9503317777	3.4557519189
1.2	1.1309733553	3.7699111843
1.3	1.3273228961	4.0840704497
1.4	1.5393804003	4.3982297150
1.5	1.7671458676	4.7123889804
1.6	2.0106192983	5.0265482457
1.7	2.2698006922	5.3407075111
1.8	2.5446900494	5.6548667765
1.9	2.8352873699	5.9690260418
2.0	3.1415926536	6.2831853072
2.1	3.4636059006	6.5973445725
2.2	3.8013271108	6.9115038379
2.3	4.1547562844	7.2256651033
2.4	4.5238934212	7.5398223686
2.5	4.9087385212	7.8539816340
2.6	5.3092915846	8.1681408993
2.7	5.7255526112	8.4823001647
2.8	6.1575216010	8.7964594301
2.9	6.6051985542	9.1106186954
3.0	7.0685834706	9.4247779608

Dia.	Area.	Circumference.
3.1	7.5476763502	9.7389372261
3.2	8.0424771932	10.0530964915
3.3	8.5529859994	10.3672557368
3.4	9.0792027689	10.6814150222
3.5	9.6211275016	10.99553742876
3.6	10.1787601976	11.3097335529
3.7	10.7521008569	11.6238928183
3.8	11.3411494795	11.9380520836
3.9	11.9459060653	12.2522013490
4.0	12.5663706144	12.5663706144
4.1	13.2025431267	12.8805298797
4.2	13.8544236923	13.1946891451
4.3	14.5120120412	13.5088484104
4.4	15.2053084423	13.8230076758
4.5	15.9043128088	14.1371669412
4.6	16.6190251375	14.4513262063
4.7	17.3494454294	14.76548554719
4.8	18.0955736847	15.0796447372
4.9	18.8574099032	15.3938040026
5.0	19.6349540849	15.7079632679
5.1	20.4282062300	16.0221226333
5.2	21.2371663383	16.3362817987
5.3	22.0618344098	16.6504410640
5.4	22.9022104447	16.9646003294
5.5	23.7582944428	17.2787595947
5.6	24.6300864041	17.5929188601
5.7	25.5175863288	17.9070781255
5.8	26.4207942167	18.2212373908
5.9	27.3397100679	18.5353966562
6.0	29.2742328822	18.8495559215

Dia.	Area.	Circumference.
6.1	29.2246656600	19.1637151869
6.2	30.1907054010	19.4778744523
6.3	31.1724531052	19.7920337176
6.4	32.1699087728	20.1061929830
6.5	33.1830724035	20.4203522483
6.6	34.2119439976	20.7345115137
6.7	35.2565235549	21.0486707791
6.8	36.3168110755	21.3628300444
6.9	37.3928065594	21.6769893098
7.0	38.4845100065	21.9911485751
7.1	39.5919214169	22.3053078405
7.2	40.7150407905	22.6194671058
7.3	41.8538681274	22.9336263712
7.4	43.0084034276	23.2477856366
7.5	44.1786466911	23.5619449019
7.6	45.3645979178	23.8761041673
7.7	46.5662571078	24.1902634326
7.8	47.7836242611	24.5044226980
7.9	49.0166993776	24.8185819734
8.0	50.2654824574	25.1327412287
8.1	51.5299735005	25.4469004941
8.2	52.8101725068	25.7610597594
8.3	54.1060794765	26.0752180248
8.4	55.4176944093	26.3893782902
8.5	56.7430173055	26.7035375555
8.6	58.0880481642	27.0176968209
8.7	59.4467869876	27.3318560862
8.8	60.8212337735	27.6460153517
8.9	62.2113885227	27.9601746169
9.0	63.6172512352	28.2743338823

Dia.	Area.	Circumference.
9.1	65.0388219109	28.5884931477
9.2	66.4761005500	28.9026524130
9.3	67.9290871522	29.2168116784
9.4	69.3977817178	29.5309709437
9.5	70.8821842466	29.8451302091
9.6	72.3822947387	30.1592894745
9.7	73.8981131941	30.4734487398
9.8	75.4296396127	30.7876080052
9.9	76.9768739946	31.1017672705
10.0	78.5398153397	31.4159265359
10.1	80.1184666482	31.7300858013
10.2	81.7128249199	32.0442450666
10.3	83.3228911548	32.3584043320
10.4	84.9486653531	32.6725635973
10.5	86.5901475146	32.9867228627
10.6	88.2473376393	33.3008821281
10.7	89.9202357274	33.6150413034
10.8	91.6088417787	33.9292006588
10.9	93.3131557933	34.2433599241
11.0	95.0331777711	34.5575191895
11.1	96.7689077122	34.8716784548
11.2	98.5203456166	35.1858377202
11.3	100.2874914842	35.4999969856
11.4	102.0703453151	35.8141562509
11.5	103.8689071093	36.1283155163
11.6	105.6831768668	36.4424747816
11.7	107.5131545875	36.7566340470
11.8	109.3588402715	37.0707933124
11.9	111.2202339187	37.3849525777
12.0	113.0973355292	37.6981118431

The Use of the foregoing Table is obvious, for

1. **I**F the Diameter of a Circle be *Inches, Feet Yards, &c.* not exceeding 12, you have it in the first Column to every tenth part of an *Inch, Foot, Yard, &c.* And against any Diameter in the first Column, you have the Area in the second Column, and the Circumference in the third: And the Area is always Square *Inches, Feet, Yards, &c.* according as the Diameter or Circumference is given in *Inches, Feet, Yards, &c.*

2. If the Diameter given be intire *Inches, Feet, Yards, &c.* not exceeding 120, you have it in the first Column, by taking away the Point, for by this means the Tenths are made Units; but in this case you must remove the Point two places towards the right Hand in the Area, and one place in the Circumference.

Thus if the Diameter

be 11.6 the { Area is 105 6831768658
Circumfer. is 36.4424747816

But if the Diameter

be 116, the { Area is 10568.31768668
Circumfer. is 364.424747816

And for the same Reason if the Diameter

be 1160, the { Area is 1056831.768668
Circumfer. is 3644.24747816

3. If any Area in the Table be multiplied by 4, the Product will be the Area of a Circle of twice the Diameter of the Circle given, and therefore the Area of a Circle whose Diameter is 22.6 (though not found in the Table) may be readily obtained; for the half of 22.6 is 11.3, which found in the first Column, the Area against it is 100.2874914842, this multiplied by 4, is 401.1499659368, the Area of a Circle whose Diameter is 22.6.

4. If the Diameter and Altitude of a *Cylinder* or *Cone* be given, multiply the Area against the Diameter by the Altitude, the Product will be the Content of the *Cylinder*; or by one third part of the Altitude, the Product will be the Content of the *Cone*.

5. If a *Cylinder*, *Sphere* and *Cone*, have the same Diameter and Altitude, their Contents are as 3, 2, 1. Suppose the Diameter and Altitude 12, the Area against 12 is 113.0973355292, this multiplied by

$$\left. \begin{array}{l} 12 \\ 8 \\ 4 \end{array} \right\} \text{ gives } \left\{ \begin{array}{l} 1357.1680263504 \\ 904.7786842336 \\ 452.3893421168 \end{array} \right\} \begin{array}{l} \text{Cylinder.} \\ \text{Sphere.} \\ \text{Cone.} \end{array}$$

So that any Area multiplied by two thirds of its Diameter, gives the Solidity of a Sphere of that Diameter.

6. If the Diameter of a Sphere be doubled, the Area against the Sum is the Superficial Content of the Sphere. As, let the Diameter be 6, the double is 12, the Area against 12, is 113.0973355292, the superficial Content of a Sphere whose Diameter is 6.

A TABLE shewing the *Area* and *Diameter* of any Circle, in *Inches*, *Foot*, *Yards*, &c. to any *Circumference* in *Inches*, *Foot*, *Yards*, &c. from 1 to 12 or from 10 to 120.

Circ.	Area.	Diameter.
1.0	.0795774715	.3183098862
1.1	.0962887406	.3501408748
1.2	.1145915590	.3819718634
1.3	.1344859269	.4138028520
1.4	.1559718442	.4456338407
1.5	.1790493110	.4774648293
1.6	.2037183272	.5092958179
1.7	.2299788928	.5411268065
1.8	.2578310078	.5729577951
1.9	.2872746723	.6047887837
2.0	.3183098862	.6366197724
2.1	.3509366495	.6684507610
2.2	.3851549623	.7002817496
2.3	.4209648245	.7321127382
2.4	.4583661361	.7639437268
2.5	.4973591972	.7957747155
2.6	.5379437077	.8276057041
2.7	.5801197676	.8594366927
2.8	.6238873769	.8912676818
2.9	.6692465357	.9230986699
3.0	.7161972439	.9549296586

Circ.	Area.	Diameter.
3.1	.7647395016	.9867606472
3.2	.8148733086	1.0185916358
3.3	.8665985651	1.0504226244
3.4	.9199155711	1.0822536130
3.5	.9748240264	1.1140846016
3.6	1.0313240312	1.1459155903
3.7	1.0894155855	1.1777465789
3.8	1.1490986891	1.2095775675
3.9	1.2103733422	1.2414085561
4.0	1.2732395447	1.2732395447
4.1	1.3376972967	1.3050705333
4.2	1.4037465981	1.3369015220
4.3	1.4713874489	1.3687325106
4.4	1.5406198491	1.4005634992
4.5	1.6114437988	1.4323944878
4.6	1.6838592979	1.4642254764
4.7	1.7578663464	1.4960564651
4.8	1.8334649444	1.5278874537
4.9	1.9106550918	1.5597184423
5.0	1.9894367886	1.5915494309
5.1	2.0698100349	1.6233801495
5.2	2.1517748306	1.6552114082
5.3	2.2353311757	1.6870423968
5.4	2.3204700703	1.7188733854
5.5	2.4072185143	1.7507043740
5.6	2.4955495077	1.7825353626
5.7	2.5854720505	1.8143663512
5.8	2.6769861428	1.8461973399
5.9	2.7700917845	1.8780283285
6.0	2.8647889757	1.9098523171

Circ.	Area.	Diameter.
6.1	2.9610777162	1.9416903057
6.2	3.0589580062	1.9735213943
6.3	3.1584298457	2.0053622830
6.4	3.2594932345	2.0371832716
6.5	3.3621481728	2.0690142602
6.6	3.4663946605	2.1008452488
6.7	3.5722326977	2.1326762374
6.8	3.6796622843	2.1645072260
6.9	3.788684203	2.1963382147
7.0	3.8992961058	2.2281692033
7.1	4.0115003406	2.2600001919
7.2	4.1252961249	2.2918311805
7.3	4.2406834587	2.3236621691
7.4	4.3576623419	2.3554931578
7.5	4.4762327745	2.3873245464
7.6	4.4963947565	2.4191551350
7.7	4.7181482880	2.4509861236
7.8	4.8414933689	2.4828171122
7.9	4.9664299992	2.5146481009
8.0	4.0929581789	2.5464790895
8.1	5.2210779081	2.5783100781
8.2	5.3507891867	2.6101419667
8.3	5.4820920148	2.6419720553
8.4	5.6149863923	2.6738030439
8.5	5.7494723192	2.7056340326
8.6	5.8855497955	2.7374650212
8.7	6.0232188213	2.7692960093
8.8	6.1624793965	2.8011269984
8.9	6.3033315212	2.8329579870
9.0	6.4457751952	2.8647889757

Circ.	Area.	Diameter.
9.1	6.5898104187	2.8966199643
9.2	6.7354371916	2.9284500529
9.3	6.8826555140	2.9602819415
9.4	7.0314653858	2.9921129301
9.5	7.1818668070	3.0239439187
9.6	7.3338597777	3.0557749074
9.7	7.4874442978	3.0876058966
9.8	7.6426203673	3.1194368846
9.9	7.7993879862	3.1512678732
10.0	7.9577471546	3.1830988618
10.1	8.1176978724	3.2149298505
10.2	8.2792401396	3.2467608391
10.3	8.4423739563	3.2785918277
10.4	8.6070993224	3.3104228163
10.5	8.7734162379	3.3422538049
10.6	8.9413247029	3.3740847935
10.7	9.110824713	3.4059157822
10.8	9.2819162811	3.4377467708
10.9	9.4545993944	3.4695777594
11.0	9.6288740571	3.5014087480
11.1	9.8047402692	3.5332397366
11.2	9.9821980307	3.5650707253
11.3	10.1612473417	3.5969017139
11.4	10.3418882021	3.6287327025
11.5	10.5241206120	3.6605636911
11.6	10.7079445712	3.6923946797
11.7	10.8933600799	3.7242256684
11.8	11.0803671381	3.7560566570
11.9	11.2689657456	3.7878876456
12.0	11.4591559026	3.8197186342

The Uses of the Table are very plain; for

1. **I**F the Circumference of a Circle be given in Inches, Foot, Yards, &c. not exceeding 12, you have it in the first Column to every tenth part of an Inch, Foot, Yards, &c. from 1, to 12, and from 10, to 120, in intire Inches, Foot, Yards, &c. and in either case you have the Area in the second Column, and the Diameter in the third Column.

Thus if the Circumference of a Circle

be 10.5, the $\left\{ \begin{array}{l} \text{Area is } 8.7734162379 \\ \text{Diam. is } 3.3422538049 \end{array} \right.$

But if the Circumference

be 105, the $\left\{ \begin{array}{l} \text{Area is } 877.34162379 \\ \text{Diam. is } 33.422538049 \end{array} \right.$

2. If any Area in this Table be multiplied by 4, you have the Area of a Circle to twice the Circumference of the Circle given. Let 8.9413247029 (the Area against 10.6 Circumference) be multiplied by 4, the Product will be 35.7652988116 which is the Area of a Circle whose Circumference is 21.2.

3. If the Circumference of a Cylinder be 10.6 Foot, and the Altitude 16 Foot, the Area against 10.6, multiplied by 16, gives the solid Content of the Cylinder in Feet, and so of any other.

4. If the Circumference of a *Globe* or *Sphere* be given, the Area against it multiplied by 4, gives the superficial Content of such a *Sphere*. Thus if the Circumference of a *Sphere* be 3 Foot, the Area against 3 is .7161972439; this multiplied by 4, gives 2.8647889756 the superficial Content of a *Sphere*, whose Circumference is 3 Foot. Or thus, double the Circumference given, and find the Sum in the first Column, the Area against it is the superficial Content sought; thus twice 3 is 6, which found in the first Column, the Area against it is 2.8647889756, as above. And indeed, any Area in this Table is equal to the superficial Content of a *Sphere*, whose Circumference is half the Circumference against such Area respectively.

5. If the Circumference of a *Sphere* be found in the first Column, you have the Diameter in the third Column. Now if the Area between them be multiplied by two thirds of the Diameter, you have the Solidity of such *Sphere*.

S E C T.

S E C T. XVII.

The Use of the Instrument in Measuring of Timber and Stone.

Of Round Timber.

Problem I.

The Circumference and Length of any Round Tree, being given (in Foot-measure) to find the Content.

R U L E.

As 3.545 upon D, is to the Length upon C; So is the Circumference upon D, to the Content upon C.

Example. Suppose the Length of a Tree be 20 Foot, and the Circumference 4 Foot, What is the Content?

Set 3.545, upon D, to 20 (the Length) upon C, then against 4 the Circumference upon D, you have 25.46 upon C, that is 25 Feet and almost an half, and so much is the Content.

Note, 3.545, is the Circumference of a Circle whose Area is 1. Pro-

Problem II.

The Circumference of a Round Tree being given in Inches, and the Length in Feet, to find the Content in Feet.

THE Rule given in the last Problem will serve here if instead of 3.545, you make use of 42.54. (which is found by multiplying 3.545 by 12.)

Example. The Circumference is 36 Inches, and the Length 10 Feet, What is the Content?

Set 42.54, upon D, to 10 upon C, and against 36 upon D, is 7.16 upon C, which is the Content sought.

Problem III.

The Circumference of any Round Tree being given, to find how much in Length it will take to make a Solid Foot.

First in Foot-Measure, the Proportion is,

As the Circumference upon D, is to 1 upon C;
So is 3.545 upon D, to the Length sought upon C.

Example. If the Circumference of a Tree be 3 Feet, How many Feet in Length will it take to make a Solid Foot?

Set

Sect. XVII. Of Round Timber. 255

Set 3 upon D, to 1 upon C, and against 3.545 upon D, is 1.396 upon C, that is, 1 Foot and .396 Parts. Now, so many times as this is contain'd in the Length of the Tree, so many Feet does the Tree contain.

Secondly, In Inch-measure, if the Circumference be given in Inches, Say,

As the Circumference upon D, is 12 upon C;
So is 42.54 upon D, to the Length in Inches (upon C) to make a Foot.

Example. The Circumference of a Tree is 48 Inches; How many Inches in Length will make a Foot?

Set 48 upon D, to 12 upon C, and against 42.54 upon D, is 9.42 upon C, that is 9 Inches and .42 Parts, the Length required.

Now, supposing this Tree were 20 Foot long, the Content will be found to be 25.47, that is, 25 Foot, and almost a half; for 20 Foot is 240 Inches, and 9.42 Inches are contain'd 25.47, times in 240.

Note, The common way used for the Measuring of Round Timber is to girt the Tree about in the middle with a Line, and to take a fourth part of the Circumference for the Side of a Square equal to the Circle; but this is false, though most Men use it; for according to this Supposition it will take 12 Inches in Length of the Tree above-mention'd to make a Foot, and so the whole Tree would contain but 20 Foot, whereas the true Content is 25.47 Foot, as was shewn above.

Note, In all that hath been said of Round Timber, we suppose the Tree to be equally thick from end to end. But seeing most Trees are less towards

wards the Top than at the Root, the Circumference must be taken in the middle, and if the Ends differ much, it will be the more exact, if you divide the Length into two equal parts, and take the Circumference in the middle of each, and so find the Content as if they were two Trees. These Contents added together, give the Content of the whole.

For Instance, There is a Tree of 12 Foot long, the Circumference at one end is 6 Foot, and at the other end 3 Foot, the Sum of these is 9, the half is 4.5, the Circumference in the middle, by which (and the Length) the Content will be found (by Problem I.) to be 19.34 Foot.

But if this Tree be divided into parts, as each may be 6 Foot long, the Circumference in the middle of the greater will be 5.25, by which the Content will be found to be 13.16 Foot.

And the Circumference in the middle of the lesser piece will be 3.75, and the Content of this will be 6.72, which added to the Content of the other, viz. 13.16, gives 19.88 Foot, the Content of the whole Tree: Whereas the Content found by one Circumference taken in the middle of the whole Tree, is but 19.34,

But in most Cases it will be sufficient to take the Circumference in the middle of the Tree, and so find the Content as in Problem I.

Observe also, in Taper-Trees, when it is said so much in length will make a Foot, it must be understood in that place only where the Circumference was taken; for it will not take so much in Length of the greater end of a Tree to make a Foot, as it will of the lesser.

Yet

Sect. XVII. Of Round Timber. 257

Yet, if by the Circumference taken in the middle you find the Length of a Foot, that Length so found will shew the whole Content of the Tree, as in Problem III.

Problem. IV.

The Circumference of any Round Tree being given, either in Feet or in Inches, to find how many Solid Feet are contained in one Foot of the Length.

IF the Circumference be given in Feet, set 3.545 upon D, to 1 upon C; then against the Circumference upon D, is the Content upon C.

Example. Let the Circumference of a Round Tree be 4 Foot, How many Solid Feet are contained in one Foot of the Length of this Tree? Answer, 1.273:

For. As 3.545 upon D, is to 1 upon C;
So is 4 upon D, to 1.273 upon C.

If the Circumference be given in Inches, make use of 42.54, instead of 3.545, and then the Proportion will hold as above.

Of

Of Square Timber.

Problem V.

The Length and Side of any Piece of Square Timber, being given in Foot-Measure, to find the Content.

The Proportion is,

As 1 upon D, is to the Length upon C;
So is the Side upon D, to the Content upon C.

Example. There is a Piece of Timber exactly Square, each side at either end being 1.5 Foot, and the Length 20 Foot, How many solid Feet doth this Piece contain?

Set 1 upon D, to 20 upon C, and against 1.5 upon D, is 45 upon C, which is the Content sought.

Problem VI.

The Side of the Square solid being given in Inches, and the Length in Feet, to find the Content in Feet.

THE Proportion in the last Problem will hold here, if instead of 1 you make use of 12 for the first Term.

Example.

Example. Let the Side be 18 Inches, and the Length 20 Foot, what is the Content in Feet?

Answer 45 : For,

As 12 upon D, is to 20 the Length upon C;
So is 18 the Side upon D, to 45 the Content upon C.

When the Ends are not exactly square (which doth often happen) take the Breadth and Thickness at the end in Feet and Parts; then multiply the Breadth by the Thickness, and this Product by the Length, the last Product will be the Content.

If the Breadth and Thickness be given in Inches, and the Length in Feet, then by the Lines A and B on the Rule,

The Proportion is,

(1.) As 1 upon B, to the Breadth upon A;

So is the Thickness upon B, to a fourth Number, viz. (the Area of the end in square Inches.)

(2.) As 144 upon B, to the fourth Number upon A;
So is the Length upon B, to the Content sought.

But if any such Solid be greater at one end than at the other, the exact way to find the Content, is to find the Area of each Base (in Foot-measure) then find a Geometrical Mean betwixt the said Area's; then if the Sum of these three be multiplied by one third part of the Length, the Product is the Content.

S E C T.

S E C T. XVIII.

The Use of the Rule in Mensuration of Brick-work.

Brick-work is commonly measured by the Rod-Square, which is 16.5 Feet in Length, and as much in Breadth, and by consequence each square Rod doth contain 272.25 square Foot; for 16.5 multiplied by 16.5 is 272.25.

Note also, all Brick-work must be reduced to the Standard-measure of one Brick and a half thick.

Problem I.

The Length and Height of any Brick-Wall being given in Feet, to find the Content in square Rods at any thickness.

The R U L E.

Multiply the Height by the Length, and divide the Product by 272.25, the Quotient will be the true Content, if the Wall be just one Brick and a half thick:

If the thickness be more or less; Then say,

As 3, is to the Number of half Bricks in the Thickness of the Wall;

So is the Quotient above found, to the true Content sought.

Or

Sect. XVIII. Of Brick-work. 263

Or having the Length and Height in Feet, you may find the Content more readily on the Rule by this Proportion.

As 272.25 upon B, is to the Height upon A ;

So is the Length upon B to a fourth Number upon A, which is the true Content, if the Wall be one Brick and a half thick : If the thickness be more or less, say,

As 3 upon B, is to the half Bricks thick upon A ;

So is the fourth Number above found upon B, to the Content required.

Example. There is a Wall 16 Foot high, and 97 Foot long : How many square Rod doth this Wall contain at one Brick and a half thick ? Answer 5 Rod and 7 tenth parts : For,

As 272.25 upon B, is to 16 the Height upon A :

So is 97 the Length upon B, to 5.7 upon A.

Now if the Wall above-mention'd were two Bricks and a half thick, it would contain 9 Rod and an half : For,

As 3 upon B, is to 5 (the half Bricks in two and a half) upon A ;

So is 5.7 the Content at one Brick and a half thick upon B, to 9.5 the Content sought upon A.

But if a Wall of 16 Foot high, and 97 Foot long were but 1 Brick thick, the Content would be but 3 Rod and 8 tenth parts : For,

As

As 3 upon B, is to 2 (the half Bricks in one) upon A;

So is 5.7 (the Content at one and a half thick) upon B. to 3.8 the Content upon A.

Now, that (in all these Cases) you may find the Content at one Operation: I shall here lay down certain fix'd Numbers for any Thickness hereafter express'd,

viz. at	1	Bricks thick	408.37
	1 $\frac{1}{2}$		272.25
	2		204.19
	2 $\frac{1}{2}$		163.35
	3		136.12
	3 $\frac{1}{2}$		116.68
	4		101.10
	4 $\frac{1}{2}$		90.75
	5		81.68

By these Numbers the Content of any Wall, at any Thickness here mentioned may be found either by the Pen or Rule, and the Proportion is,

As the Number proper to the Thickness, is to the Height;

So is the Length to the Content.

Example. There is a Wall four Bricks and a half thick, 7 Foot high, and 85 Foot long: How many square Rod doth this Wall contain?

The proper Number for four Bricks and a half thick, is 90.75.

Therefore,

Therefore, Set 90.75 upon A, to 7 the Height upon B; then against 84 the Length upon A, is 6.555 upon B; that is, six Rod and a half, and something more.

Note, The Decimal parts of a Rod may be reduced into Feet by this Proportion.

As 1.00 upon A, is to 272.25 upon B;

So is any Decimal upon A, to the Number of Feet upon B.

And thus you may find the Decimal in the last Example, viz. .555 is equal to 151 Foot.

Also $\left\{ \begin{array}{l} \frac{1}{4} \text{ or } .25 \\ \frac{1}{2} \text{ or } .50 \\ \frac{3}{4} \text{ or } .75 \end{array} \right\}$ of a Rod $\left\{ \begin{array}{l} 68 \\ 36 \\ 20\frac{1}{2} \end{array} \right\}$ Feet

Problem. II.

The Height of any Wall being given in Feet, to find how many Feet in Length it will take to make a Square Rod at any Thickness abovemention'd.

R U L E.

SET the Height upon A, to the Number proper for the Thickness upon B; then against 1 upon A, is the Length sought upon B.

Example

Example. There is a Wall one Brick and half thick, and 8 Foot high: How many Feet in Length will it take to make a square Rod? Answer, 34.

For,

the proper Number for one and a half thick is 272.25. Therefore,

As 8 (the Height) upon A, is to 272.25 upon B;
So is 1 upon A, to 34 upon B;

S E C T. XIX.

The Use of the Rule in the Mensuration of Superficies, as Board, Paving, Cieling, Wainscoting, Flooring, Tyling, &c.

Note, In all Square or Oblong Superficies the Area or Content may be found by this Proportion :

As 1 upon A, to the Breadth upon B.

So is the Length, upon A, to the Content upon B.

This Proportion will hold in Triangular Superficies also, if you take half the Perpendicular for the Breadth, and the Side upon which it falls for the Length: And in all Cases, if the Sides be given

in { Inches } the Area { Inches, }
 { Feet } will be { Feet or }
 { Yards } Square { Yards. }

But

Sect. XIX. Of Board Measure. 265

But in many Cases the Sides are given in one kind of Measure, and the Area required in another :
I shall therefore give some Examples of each.

I. For Board.

IF the Length and Breadth be both given in Inches, and the Content sought in Feet, Say,

As 144, is to the Breadth ;
So is the Length, to the Content.

Example. A Board is 14 Inches broad, and 156 Inches long: How many square Feet doth it contain ? Answer 15.16. For,

As 144 upon A, is to 14 the Breadth upon B ;
So is 156 the Length upon A, to 15.16 the Content upon B ;

If the Breadth be given in Inches, and the Length in Feet, Say,

As 12, is to the Breadth ;
So is the Length, to the Content.

Example. A Board is 14 Inches broad, and 13 Foot long: What is the Content in Feet ? Answer, 15.16. For,

As 12 upon B, is to 14 the Breadth upon A ;
So is 13 the Length upon B, to 15.16 the Content upon A.

Of Paving.

PAvier's Work is measured by the Yard Square, which contains 9 square Feet: The most natural way is to measure the Sides with a Yard divided into 100 equal parts, and then multiply the Breadth by the Length, the Product will be the Content in square Yards: But if the Sides be given in Feet, say,

As 9 is to the Breadth;
So is the Length, to the Content.

Example. There is a Court pay'd with Stone, the Breadth is 64 Foot, and the Length 95 Foot: What is the Content in square Yards? Answer, 67.55. For,

As 9 upon A, is to 64 the Breadth upon B;
So is 95 the Length upon A, to 67.55 the Content upon B.

III. Of Cieling.

Cieling and also Wainscoting, are measured by the Yard Square, and therefore what is said in the Example above, may be understood here.

IV. Of Flooring and Tiling.

THese are both measured by the Square, which contains 100 Square Foot, the Side being 10 Foot : Therefore if the Length and Breadth be given in Feet, multiply one by the other, and cut off two Figures in the Product, the Remainder shews the Content in Squares, and the Figures cut off are the parts of a Square, or odd Feet.

Or by the Rule, say,

As 100, is to the Breadth in Feet ;

So is the Length in Feet, to the Content in Squares.

Example. There is a Floor 66 Foot broad, and 75 Foot long. How many Squares doth it contain ? Answer, 49.5. For,

As 100 upon A, is to 66 the Breadth upon B ;
So is 75 the Length upon A, to 49.5 the Content upon B.

The like for Tiling.

Lastly, The Breadth of any Superficies being given in Feet or Yards, to find how many in Length (of either) it will take to make a Foot or Yard square.

The Proportion is,

As the Breadth is to 1 ;

So is 1, to the Length sought.

N 2

Example.

268 *Of Flooring and Tying.* Sect. XIX.

Example. A Board is 1.5 Foot broad, : How much in Length will it take to make a Foot square ? Answer, .667. For,

As 1.5 (the Breadth upon) A, is to 1 upon B;
So is 1 upon A, to .667 upon B.

So that if one side of a piece of Board be 1.5 Foot, the other side must be .667 of a Foot, to make a square Foot ; for .667 multiplied by 1.5 is 1.

In Inch-measure the Proportion is,

As the Breadth in Inches, is, to 12 ;

So is 12

To the Length in Inches to make a Foot.

And, As the Breadth in Inches is to 36,

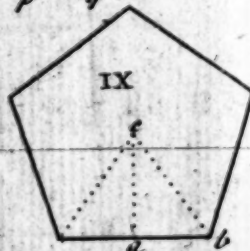
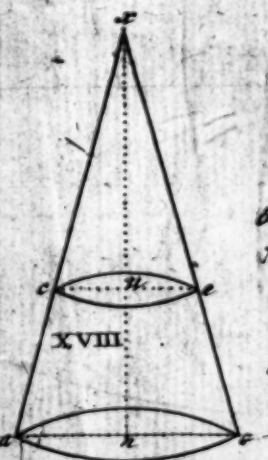
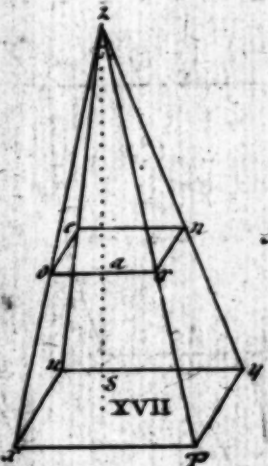
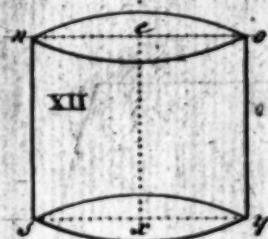
So is 36

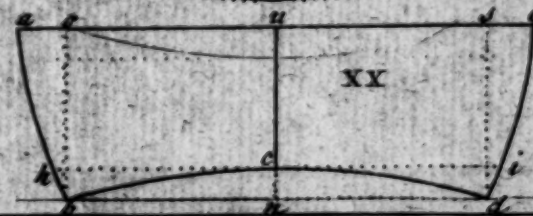
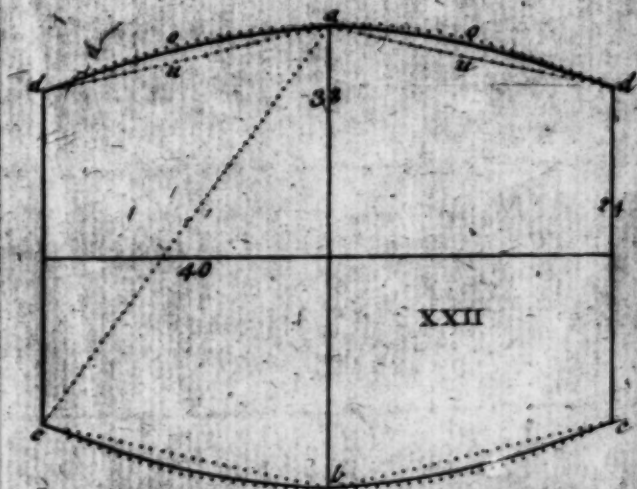
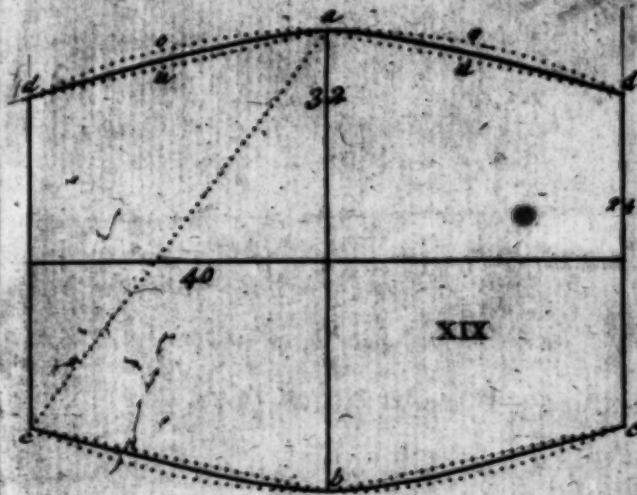
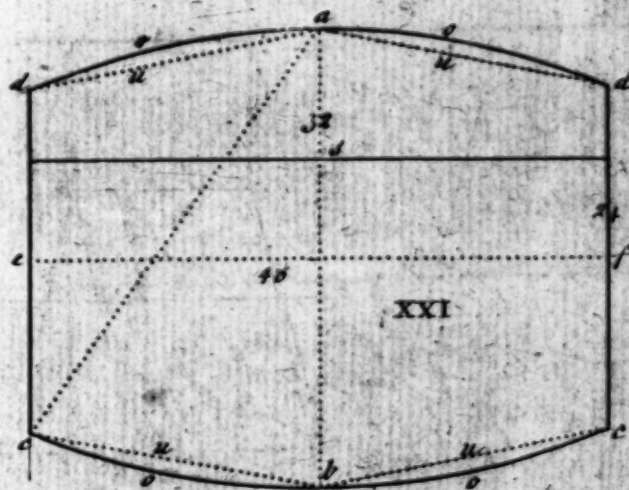
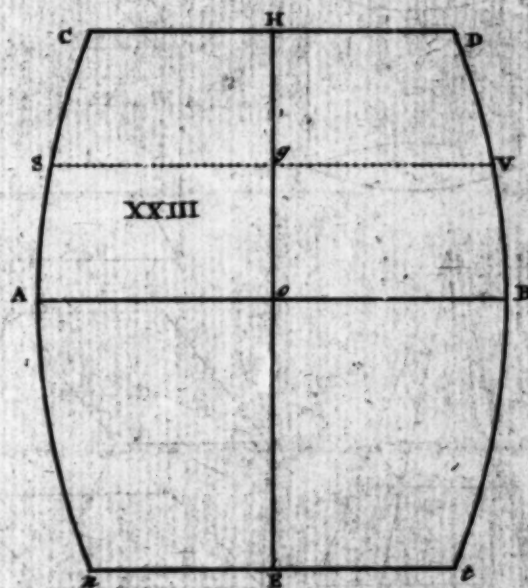
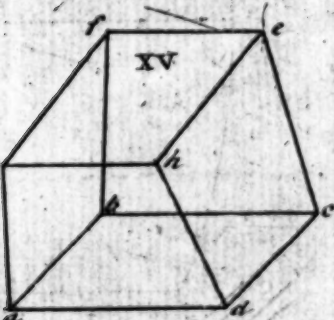
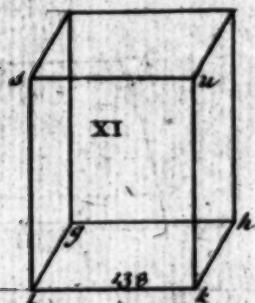
To the Length in Inches to make a Yard.

Example. A Board is 18 Inches broad, how many Inches in Length will make a Foot ?

Set 18 (the Breadth) upon A, to 12 upon B, then against 12 upon A, is 8 upon B ; and so many Inches in Length will make a Foot of this Board ; for 18 multiplied by 8, gives 144 the square Inches in a square Foot.

A P P E N.





Place this before & Appendix.

APPENDIX.

Of Conical Sections.

IN the Doctrine of *Conicks* there are these two things (very different) to be separately considered: *First*, What Figures or Curve-Lines do arise from the Section of a Cone by a Plain. And *Secondly*, See Dr. Wallis *de Con. Sect.* and What is the nature of such Figures *Treatise of Algebra*, pag. 292. or Curve-Lines, considered without respect had to such supposed Original, by cutting a Cone.

DEFINITION.

1. A Cone is a Solid described, when one of the containing Sides of a Rectangled Triangle remaining fixed, the Triangle is turned round about till it return to the place from whence it first moved: The Axis of a Cone is that fixed Side about which the Triangle is moved: The Base of the Cone is a Circle, which is described by the containing Side that is moved about.

Of the several Sections of a Cone.

1. IF a Cone be cut by a Plain through its Axis, from the Vertex to the Center of the Base. the Section will be a *Triangle* as SVB. Fig. 1 and 2.

2. If a Cone be cut by a Plain at Right Angles to its Axis, (or if a Right Cone, Parallel to its Base) the Section will be a *Circle*, whose Diameter let be S d B, Fig. 1.

3. If a Cone be cut by a Plain Parallel to the side of the Triangle (SV) the Crooked Section C c c N c c C, will be a *Parabola*; in which PN, is the Diameter, or Axis, and d c an ordinately applied Line. Vid. Fig. 1.

4. If a Cone be cut by a Plain that will also cut one of the sides of the Triangle produced in E, (Fig. 1.) the Crooked Section O o o A o o O, is an *Hyperbola*, in which A H is the intercepted Diameter: A E the Transverse Diameter; and b o, an ordinately applied Line.

5. If a Cone be cut by a Plain through both the sides of the Triangle under the Vertex V, the Crooked Section, a o o A o o will be an *Ellipsis*, in which a A is the Diameter, and D o, an ordinately applied Line. Vid. Fig. 2.

These are all the Sections that can be made by cutting a Cone by a Plain. I shall treat only of the four last, and of these but so much as I conceive necessary to my present Design, which is only

only to explain the Nature and Original of some Solids which are frequently mentioned in Books, and do sometime occur in the Practice of Gauging.

Of a Parabola.

IN the Parabola (Fig. 1.) Let the Cone be cut by a Plain (S d B) parallel to the Circular Base (S P B) it will appear,

1. That S d is equal to S P, for N P is parallel to S V, and S d B, is parallel to S P B.
2. That S P multiplied by P B is equal to P C squared. Therefore S d ~~and B~~ d c squared.
3. ~~As~~ N P, is to P C squared, So is N d, to d c squared. Therefore,
4. ~~As~~ N P, is to N d; So is P C squared, to d c squared, which is the general Property of a Parabola.

Problem I. Figure 3.

To describe a Parabola in Plano.

DRAW the straight Line O P O, and upon the middle Point P, erect the Perpendicular P A; compleat the Triangle by drawing the Lines A O, A O; then through A P (the Axis) and parallel to O P O (the Base) draw a competent number (the more the better) of straight

N 4

Lines

Lines SDS. Now wheresoever D be taken, it will be

$$AP : PO :: AD : De. \text{ And } PO \times De = D o q.$$

This gives the Points o.o (as many as you please on both sides) through which the Curve Line O o o A o o O being drawn, you have the *Parabola* required.

Or thus ;

Having drawn the *Parallelogram* ONVO, (Fig. 3.) divide AN into any number of equal parts, and suppose AP divided into as many equal parts as the square of the number of parts in AN, as here AN is divided into 4 equal parts, and AP is supposed to be divided into 16 equal Parts. Let Lines be drawn from N n.n.n. parallel to AP. Now, if from a Scale of equal parts, of which 16 are equal to AP, you make the first $nc = 1$. the second $nc = 4$. the third $nc = 9$. and the fourth $NO = 16$. the points C. C. C. O. are in the *Parabola*, and by a competent number of points thus found, the *Parabola* may be drawn.

Note, 1. The *Parabola* OcAO, is two thirds of ONVO, the *Parallelogram* circumscribed, and the Triangle OAO, is three fourths of the *Parabola*; therefore if the *Parallelogram* be 3, the *Parabola* is 2, and the *Triangle* 1.

2. If the *Parallelogram* PANO, be turned about the Line AP, it will generate the *Cylinder* ONVO,

ONVO, and by the same conversion there will be generated by the Curve-Line O c c c. the *Parabolick Conoid*, OCAOO, and by the streight Line OA, the Cone O A O, and these three Solids are as 3. 1 and half, and 1; for the *Parabolick Conoid*, is half the circumscribing *Cylinder*, and the Cone one third part thereof. Now, all these properties being common to every *Parabola* (as is demonstrated by Dr. Wallis in his *Mechanica*, Part 2 Chap. 5.) the Content of any *Parabolick Conoid* is easily found, as shall be shewn in the next Problem.

Problem II.

There is a *Parabolick Conoid* (Fig. 3.) O A O, the Diameter of the Base is 32 Inches, and A P, the Altitude 45.7143 Inches: What is the Content in Ale-Gallons?

Multiply the Area of the Base by half the Altitude, the Product gives the Content sought.

Example. The Diameter is 32. The Area is 2.8519, this multiplied by half the Altitude (*viz.*) 22.8571 gives 65.18 the Content sought.

Or, Multiply the Square of the Diameter by half the Altitude, and that Product by .0027851, the last Product will be the Content in Gallons.

Problem III.

There is a Frustum of a Parabolick Conoid, (Fig. 3.) whose Bases are Parallel; OPO, the Diameter of the greater Bases is 32 Inches: CDO, the Diameter of the lesser Base 24 Inches, and PD the Altitude 20 Inches: What is the Content in Ale-Gallons?

From the square of PO (the Semidiameter of the greater Base) subtract the square DO (the Semidiameter of the lesser Base) then say,

As that Difference,

Is to PD the Altitude of the Frustum;

So is the Square of PO,

To PA the Axis of the whole Conoid.

In this Example it will be:

$$PO^2 - DO^2 : PD :: PO^2 : PA$$

$$112 : 20 :: 256 : 45.7143$$

Now, if from PA you subtract PD, you have the Altitude of the Parabolick Conoid, whose Base is CDO. And since every Parabolick Conoid is equal to half a Cylinder of the same Base and Altitude, the Area, of the Base OPO multiplied by half the Altitude PA, gives the Content of the Parabolick Conoid, whose Base is OPO, and Altitude A P. And the Area of the Base CDO, multiplied by half the

the Altitude DA , gives the Content of the *Parabolical Conoid* whose Base is CDO , and Altitude DA . Therefore, if from the Content of the former you subtract the Content of the latter you have the Content of the Frustum CDO, OPO .

Example. CDO is 24 Inches, the Area is 1.6042, this multiplied by half DA (*viz.*) 12.8571, gives 20.62 Gallons, the Content of the *Parabolical Conoid* $DCCAOO$, this subtracted from 65.18 (the Content of the *Parabolical Conoid* $OCCAOO$) leaves 44.56 Gallons, the Content of the Frustum (CDO, OPO), which was demanded. But the Content of any such Frustum may be more readily found, thus,

To the Area of the greater Base, add the Area of the lesser Base, the Sum multiplied by half the Altitude, gives the Content.

Example. In the Frustum last mentioned,

The Diameter OPO , being 32 Inches, the Area is 78.519

The Diameter CDO , being 14 Inches, the Area is 1.6042

Sum is 80.1232

This multiplied by half the Altitude (*viz.* 10.) gives 40.0616 Gallons the Content, as by the last.

Now

Now, Suppose the Fruustum OBGO, be equal to the Fruustum OCDO, and the Base OPO, common to both, this is what several Writers would have to represent a common Cask, viz. a Butt, Hog-head, Barrel, &c. But it is plain from the nature and construction of the Figure, that no such Cask can be in the Form of these two Frustrums abutting upon the common Base OPO. But if it were practicable to make a Cask in this Form, if the Diameters, and the Length be as above-mentioned, the Content will be 89.122 Gallons.

To find the Content upon every Inch of the Fruustum. CDO. OPO.

From the Square of (OPO) the Diameter of the greater Base, subtract the Square of (CDO) the Diameter of the lesser Base, and divide this difference by DP the Depth, the Quotient is the common Difference of the Squares of the several Diameters, taken one in the middle of every Inch of DP. But for the Diameter in the middle of the first Inch next the lesser Base, take half this common Difference, and add it to the Square of CDO, the Sum is the Square of the Diameter in the middle of that Inch; to this add the common Difference, the Sum will be the Square of the Diameter in the middle of the second Inch, and so of the rest. But having got the Squares of the Diameters in the middle of the first, second, and third Inch, find the Area's of those Circles in Gallons, the Difference of these Area's, will be equal.

Example.

APPENDIX

277

Example.

$$O P O = 32 \text{ Square} = 1024$$

$$C D O = 24 \text{ Square} = 576$$

$$\text{Difference} = 448$$

448 divided by D P = 20, is 22.4 the common Difference.

$$\begin{aligned} \text{Square of } C D O &= 576 \\ \text{half com. differ.} &= 11.2 \end{aligned}$$

$$\begin{aligned} \text{Square of the Diameter in the } \left. \begin{array}{l} \text{middle of the first Inch} \\ \text{Common Difference} \end{array} \right\} &= 587.2 \\ &= 22.4 \end{aligned}$$

$$\begin{aligned} \text{Square of the Diameter in the } \left. \begin{array}{l} \text{middle of the second Inch} \\ \text{Common Difference} \end{array} \right\} &= 609.6 \\ &= 22.4 \end{aligned}$$

$$\text{Square of the third Diameter} = 632.0$$

The Square of these Diameters thus found, multiplied severally by .002785, give the Area or Content of the first, second, and third Inch.

	Area's,	Diff.
<i>Viz.</i> { }	.6354	.0624
{ }	1.698	.0624
{ }	1.7602	

And so of the rest, as in the following Table, in the last Column of which are the common Difference of the Area's. In the third Column, you have the Area or Content of every Inch. The first

first Number in the second Column, viz. 44.5640 is the Sum of all the Area's in the third Column, that is, the Content of the whole Tun: From which the several Area's continually subtracted, leaves the number of Gallons contained in the Tun at any Depth. Thus against 10 Inches deep, you have 25.402 Gall. in the Tun, and so for any other Depth.

Inches of the Tun's depth.	Content at any depth.	Area's.	Com. Diffe.
20	44.5640	1.6354	.0624
19	42.9286	1.6978	.0624
18	41.2308	1.7602	.0624
17	39.4706	1.8226	.0624
16	37.6486	1.8850	.0624
15	35.7630	1.9474	.0624
14	33.8156	2.0098	.0624
13	31.8058	2.0722	.0624
12	29.7336	2.1346	.0624
11	27.5990	2.1970	.0624
10	25.4020	2.2594	.0624
9	23.1426	2.3218	.0624
8	20.8208	2.3842	.0624
7	18.4366	2.4466	.0624
6	15.9900	2.5090	.0624
5	13.4810	2.5714	.0624
4	10.9096	2.6338	.0624
3	8.2758	2.6962	.0624
2	5.5796	2.7586	.0624
1	2.8210	2.8210	.0624
Content=44.5640			

Of a Parabolick Spindle.

IN Fig. 5. RH is 40, $RS = bE$ 20. AR 16,
 Rb 12, and bH 4. And
 $AR:RHq::Ah:bEq.$
 $16:1600::4:400.$

Therefore (the other side being made like to this) the Curve Line $GCAEH$ is a *Parabola*. Now if this *Parabola* be turned about upon the Line GH , it will generate the Solid $GAHBG$, which is called a *Parabolick Spindle*. And every *Parabolick Spindle* is to its circumscribing Cylinder, as 8 to 15. (*Vide. Dr. Wallis's Mechanica, Part II. Chap. 5, Prob. 12.*)

Problem IV. Figure 5.

To describe a *Parabolick Spindle*, the middle Frustum whereof shall Represent a Cask whose Diameter at the Bung AB , is 32 Inches, the Diameters, at the Heads DC , and EF , are each 24 Inches, and the Length PS 40 Inches. And to find the Content of the middle Frustum thereof, representing the Cask proposed.

1. **D**RAW the straight Line zz , and at the right Angles to it, draw the Lines AB , CD , and EF , so as RP and RS , be each = 20.
 Make

Make $AR = 16$. CP , bR and Es , each $= 12$, so shall $Ab, be = 4$. And Because it is a *Parabola*, it will be

$$Ab : AR :: bEq : RHq$$

$$4 : 16 :: 400 : 1600$$

The Square-Root of 1600, is 40, $= RH$, or RG .

2. To find a Diameter in the middle of every Inch of RG , or RH . Through the Point A , draw IK Parallel, and equal to the Line HG , let IA and GR , be each divided into half Inches, which here will be 80, the Square of this is 6400, by which divide AR 16, the Quotient will be .0025, this multiplied by the Squares of all the odd Numbers, viz: 1, 3, 5, 7, &c. to 79, the several Products give the Length of the Line ne , which taken from the Line AR , leaves the Lines eo , the Semidiameters of the *Parabolick Spindle*, (one in the middle of every Inch of GR .) But to contract this Work, we need but the three first Diameters, the rest are obtained by subtracting the common Difference; as will appear by the following Operation.

$$\begin{array}{c} .0025 \\ \text{Multipli-} \\ \text{ed by} \end{array} \left\{ \begin{array}{c} 1 \\ 9 \\ -5 \end{array} \right\} \text{ is } \left\{ \begin{array}{c} .0025 \\ .0225 \\ .0625 \end{array} \right\} \text{ Each of}$$

these taken from AR 16, the three Remainders will

$$\begin{array}{c} \text{be} \left\{ \begin{array}{c} 15.9975 \\ 15.9775 \\ 15.9375 \end{array} \right\} \text{ these mul-} \\ \text{plied by 2} \left\{ \begin{array}{c} 31.995 \\ 31.95 \\ 31.875 \end{array} \right\} \text{ the} \\ \text{give} \text{ ————— } \end{array}$$

three

APPENDIX.

281

three Diameters, one in the middle of every Inch next to A B; to find the common Difference, take the second Diameter from the first, there rests .040; take the third from the second, the Remainder is .080; take .040 from .080, there rests .040, the second Difference, which is the same throughout; and by this common Difference the rest of the Diameters are thus found. Take the first Difference (*viz.* .040) from the first Diameter, the Remainder is the second Diameter; add the second Difference to the first Difference, and subtract their Sum from the second Diameter, the Remainder will be the third Diameter; and so of the rest, as in the following Table; in the second Column of which you have the second Difference (which is the same throughout) in the third Column are the first Differences, in the fourth Column you have the true Diameters of the Spindle, one in the middle of every Inch of R P. and in the last Column are the Areas to these Diameters, that is, the Content of so many circular Plains of one Inch thick, which added together make 48.249 Ale-Gallons, the Content of the Part B D C A, this doubled is 96.468, the Content of the middle Frustum CAEFBD, representing the Cask proposed.

Note. The Area's in this Table were found by multiplying the Squares of each Diameter by .00785. But this Work may be much shorten'd; for if the Squares of the several Diameters in the fourth Column be added together, and their Sum multiplied by .00785, the Product will be 48.349 = the Sum of the Area's in the last Column.

A Table shewing the Content of half the middle Frustum of the *Parabolick Spindle*, CAEFBD. Fig. 5.

	Second Differ.	First Difference	Diameter in the middle of every Inch of R. P.	Area in Gallons, or the Content of every Inch from R. to P.
1			31.995	2.851
2	40	40	31.955	2.844
3	40	0	31.875	2.830
4	40	120	31.755	2.808
5	40	160	31.595	2.780
6	40	200	31.395	2.745
7	40	240	31.155	2.703
8	40	280	30.875	2.655
9	40	320	30.555	2.600
10	40	360	30.195	2.539
11	40	400	29.795	2.473
12	40	440	29.355	2.400
13	40	480	28.875	2.322
14	40	520	28.355	2.239
15	40	560	27.705	2.152
16	40	600	27.195	2.060
17	40	640	26.555	1.964
18	40	680	25.875	1.865
19	40	720	25.155	1.762
20	40	760	24.395	1.657
Sum				48.249
Content of the Cask				96.498

Note,

Note, If the part ABDC were a Tun standing upon its lesser Base DC, and from the whole Content thereof viz. 48.249, you subtract the first Area, viz. 2.851, the Remainder 45.398, will be the Content at 19 Inches deep; and if from this you take the second Area, you will have 42.554, the Content at 18 Inches deep, and proceeding with the rest in the same manner, you will have a Table shewing the Content at any Depth.

Of an Hyperbola.

IN Fig. 1. the Cone SVB, is cut by a Plain HA, which also cuts one Side of the Triangle produced in E, therefore the crooked Section Ooo A o o O, is an *Hyperbola*. In which EA is the transverse Diameter, and AH the intercepted Diameter: Now if the Cone be cut by an other plain Parallel to the Base, this Section will be a Circle, whose Diameter is SHB: And $SH \times HB = HOq$, therefore HO is an ordinately applied Line, and if HO be a Line ordinately applied, it will be

$$AH \times EH : Ab \times Eb :: HOq : boq :$$

Pro-

Problem V. Fig. 4

To find the Content of an Hyperbolick Conoid, or the Content of any part thereof in Ale-Gallons.

IF the *Semi-Hyperbola* ASSD, be turned round about upon the Line A D, it will describe the *Hyperbolick Conoid*, S S A o O. Now, if we put the Transverse Diameter = t , the Intercepted Diameter = d , the Magnitude of the *Hyperbolick Conoid* = A, and the Magnitude of a Cylinder of the same Base and Altitude = B, it will be

$$6t + 6d : 3t + 2d :: B : A.$$

That is, (1.) To 6 times the Transverse Diameter A B, add 6 times the intercepted Diameter A D, the Sum is your Divisor.

(3.) To 3 times the Transverse Diameter add twice the Intercepted Diameter, by this Sum multiply the Content of the Cylinder N V O S, the Product is your Dividend; which divided by the Divisor above found, the Quotient will be the Content of the *Hyperbolick Conoid*, whose Base SDO, is the Base of the Cylinder NVOS, and Altitude AD equal to S N the Altitude of the Cylinder; and this will hold wheresoever the Base be taken: For suppose the first S d o,) next the Vertex A, be taken for the Base of the *Hyperbolick Conoid*, S A o, (and of the Cylinder S n n o,) the Content of this *Hyperbolick Conoid*, may be found as above, and the Content of any Frustum, S D O o D S may be likewise found,

found; for if the little *Hyperbolick Conoid* S A o d, be taken from the *Hyperbolick Conoid* S S S w o o O, there remains the *Frustum* S D O o d s, and thus the Content of the whole, and also the Content of any *Frustum*, and consequently of every Inch of the *Hyperbolick Conoid* may be readily found, as will appear in the following Example.

Example. Fig. IV.

Let A E, the Transverse Diameter, and A D, the Intercepted Diameter, be each 32 Inches, and S D O the Diameter of the Base 45.255 Inches, to find the Content of the *Hyperbolick Conoid* in Ale-Gallons.

1. Six times A E is 192, and 6 times A D is 192, the Sum of these is 384, for the first Term of the Proportion.

2. Three times A E is 96, and twice A D is 64, the Sum of these is 160, for the second Term.

3. The Diameter of the Base S D O is 45.255 (equal the Square-Root of 2048) and the Area of this Circle is 8.704 Gallons; this multiplied by 32 (equal the Altitude S N) gives 182.528 the Content of the Cylinder N S O V, and this is the third Term of the Proportion, which will stand thus;

$$384 : 160 :: 182.528.$$

Now multiply the *third* Term by the *second* and divide that Product by the *first*, the Quotient or *fourth* Term will be 76.053 Ale-Gallons, which is the Content of the *Hyperbolick Conoid* required.

And by the same Method the Content of the little *Hyperbolick Conoid*, whose Altitude d A, is 4 Inches, and the Diameter of its Base (S d o) 16.97 Inches,

Inches, will be found to be 1545 Gallons. This taken from the Content of the Hyperbolick Conoid above-found, the Remainder is 74508 Ale-Gallons, the Content of the Fruustum S D O o d S; and so for any other Fruustum.

Problem. VI. Figure 6.

To describe an Hyperbola in Plano Geometrically.

Draw a straight Line R R, and from the middle Point W, let fall the Perpendicular W B, then divide W R in any number of Parts, and draw the Lines r o G, r o H parallel to W B, then set one Point of your Compasses in the Line W B, suppose at A, and extend the other to r, where keep it fast whilst the first Point, with the same Extent, is removed from A, to the Line r G, it will fall upon the Point O, and so on the other side. I say, the Points o o o, are in the Hyperbola, and by a competent Number of such Points thus found, the Hyperbola may be drawn. W A is half the Transverse Diameter, A B the intercepted Diameter, and B o an ordinately applied Line, (*Vid. Dr Wallis's Mechanica, Part II, Page 553.*)

Note, If the Hyperbola O o A o O, be turned about upon the Line z z, it will describe a Solid O o A o o O F D E O, which may be called an Hyperbolick Spindle.

Problem

Problem VII. Fig. 6.

To describe an Hyperbolick Spindle, the middle Frustum of which shall represent a Cask whose Diameters at the Bung A D, is 22 Inches, the Diameters at the Hands, O E, and O F, are each 24 Inches, and the Length G H, 40 Inches, and to find the Content of this Frustum in Ale-Gallons.

Draw the right Line $z z$, and at Right Angles thereto, the Line $W D$, make $B G$ and $B H$, each equal to 20 (the Semi-length of the Cask proposed) through the Points G and H , draw the Lines $r E$, and $r F$ parallel to $W D$, make $G O$ and $H o$, each equal to 12, and $B A$ 16, then make $B b$ equal to $G o$, so shall $A b$ be 4. I say, the Square $b o$, (400) divided by $A b$, (4) is equal to 100, from which subtract 4, the Remainder (96) is the Transverse Diameter of this Hyperbola, make $A W$ equal to 48, (the half of 96) and through the Point W , draw the Line $R R$ parallel to $z z$, and let the Line $r o$ be parallel to $W D$, I say, the Square of $W A$, more the Square of $W r$, is equal to the Square of $r o$, of which the Square-Root is the Length of the Line $r o$. Now the Line $r o$, taken from the Line $W B$, the Remainder is the Line $o g$, which doubled give $o e$, a Diameter in the Spindle, and this will hold wheresoever r be taken, and To innumerable Diameters $o e$, may be found on both Sides $A D$, to the ends of the Spindle. But for our present purpose, it will be sufficient to find

find a Diameter in the middle of every Inch of the part A D E O, as suppose the first Diameter o g e, be half an Inch from A D, the first W r is .5, the Square of this (*viz.* .25.) added to the Square of W A (2304) is equal to the Square of r o, (2304.25) the second W r, will be 1.5, the third 2.5, &c.) the Squares of which you have in the first Column of the following Table. In the second Column are the Sums of the Squares of W A and W r, that is, the Squares of r o. In the third Column are the square Roots of the Numbers in the second Column, that is, the Length of the Lines r o. Now each of these Numbers taken from W B 64, leave the Lines o g, which doubled are the Diameters o e, as in the fourth Column. In the last Column are the Area's of these Circles in Ale-Gallons, that is, the Content of so many Circular Plains of one Inch thick, the Sum of these is 49.1 (equal the part ADEO) which doubled is 98.2, equal to the middle Frustum O E D F O A. The Content here found is not Mathematically True, but near enough for any ordinary Practice; and by the Method here laid down, the Content may be found so near the Truth, that the difference shall be less than any quantity assignable.

Now, Let the part D E O A, represent a Tun, and suppose it to stand upon the lesser Base E o. If from the whole Content thereof, *viz.* 48.1, you subtract the first Area, *viz.* 2.851, the Remainder 45.249, will be the Content at 19 Inches deep; and if from this you subtract the second Area, *viz.* 2.843, you will have 42.406, the Content at 18 Inches deep; and proceeding with the rest after the same manner, you will have a Table shewing the Content of this Tun at any depth.

APPENDIX.

289

A TABLE shewing the Content of every Inch of half the middle Frustum of the Hyperbolick Spindle - E o A o F D (Fig. VI.) in Ale-Gallons.

Squares of W r.	Sum of the Squares of W A. and W r.	Square Roots of the Square of (r o) or the length of the Lines r o,	Diameter o e	Area's of the Circle in Ale-Gallons.
.25	2304.25	48.003	31.995	2.851
2.25	2306.25	48.023	31.953	2.843
6.25	2310.25	48.065	31.870	2.829
12.25	2316.25	48.127	31.745	2.807
20.25	2324.25	48.210	31.579	2.777
30.25	2334.25	48.314	31.372	2.741
42.25	2346.25	48.438	31.124	2.698
56.25	2360.25	48.582	30.835	2.648
72.25	2376.25	48.747	30.506	2.592
90.25	2394.25	48.931	30.138	2.520
110.25	2414.25	49.135	29.730	2.462
132.25	2436.25	49.358	29.283	2.388
156.25	2460.25	49.601	28.798	2.310
182.25	2486.25	49.862	28.275	2.227
210.25	2514.25	50.142	27.715	2.139
240.25	2544.25	50.441	27.119	2.048
272.25	2576.25	50.757	26.486	1.954
306.25	2610.25	51.090	25.819	1.857
342.25	2646.25	51.442	25.117	1.753
380.25	2684.25	51.809	24.380	1.646
Sum is				48.100
This doubled is				96.200
The Content of the Frustum E o A o F D.				

O

of

Of an Ellipsis.

IF the Cone $S e B d V$ (Fig. 2.) be cut by a Plain ($A a$) through both the Sides of the Triangle, the crooked Section $a o o A o o$ will be an Ellipsis, in which $A a$, is the Transverse Diameter, or Axis, and $A D$, a Diameter intercepted.

Let the Cone be cut by another Plain ($S B$) parallel to the Base $S e B d$, it will be $S D X B D = D o q$, and this will hold wheresoever $D b e$ is taken, and give the Lines $D o$, which are called Ordinately applied Lines, and the Lines $D o$ (cutting the Line $a A$, at Right Angles) are called Conjugate Diameters of the Ellipsis.

Problem VII. Fig. 7.

Given the Axis of the Ellipsis, and its greatest Conjugate Diameter (cutting the Axis at Right Angles) to describe the Ellipsis.

DR A W, a straight Line, and make AC , equal to the Axis, given, through the mid-
dle Point B , and at Right Angles to AC , draw the Line FL , and from the Point B , set off half the Conjugate Diameter to O and G , draw the Line, $D H$, $d h$, parallel to FL . Join $B D$, and $B d$, upon the Center B , and with the interval $B A$, describe the Circle $F A L C$. I say, $B D q \dots B E q = E D q$. And $B F : B o :: E D : E O$. This will hold wheresoever E be taken, and give the
Points

Points o o, (as many as you please on both Sides) through which the *Ellipsis* may be drawn.

Of a Spheroid. Fig. VII.

IF the *Semi-Ellipsis* A O C, be turn'd about upon the Line A C, it will generate a Solid AGCO, which is call'd a *Spheroid*.

Note, 1. The Content of every *Spheroïd* is equal to two thirds of the Content of a Cylinder, whose Base is equal to the greatest Circle of the *Spheroid*; and its Altitude equal to the *Spheroid's* Axis. Therefore the Area of the greatest Circle of a *Spheroid*, multiplied by two thirds of its Axis gives the Content.

2. If the *Spheroid*, be cut by a Plain Z Z, parallel to the Line O G, and through the middle of B C, the Content of the Frustum Z C Z, to the Content of a Cylinder, whose Base is Z Z, and Altitude V C, is as 5 to 9. Therefore, the Area of the Base Z Z, multiplied by five Ninths of V C, gives the Content of the Frustum Z C Z.

3. If the *Spheroid* be cut as above, the upper Frustum Z O A H G Z, is equal to a Cylinder, whose Base is Z Z, and its Altitude V A. Therefore the Area of the Base Z Z, multiplied by the Altitude V A, gives the Content of the Frustum Z O A G Z. These three Propositions do also hold in a *Sphere*. Vid. Dr. Wallis's *Treatise of Algebra*, Page 312.

Problem VIII. Fig. 7.

To describe a Spheroid, the middle Frustum of which shall represent a Cask, whose Bung-Diameter OG , is 32 Inches, the Diameters at the Heads OH and OK , are each 24 Inches, and the Length EE 40 Inches. And to find the Content of such Cask upon every Inch of the Depth $E E$.

From the Square of (OG) the Bung-Diameter, Subtract the Square (OH) the Head-Diameter. Then.

*As that Difference,
Is to the Square of OG ,
So is the Square of BE ,
To the Square of BA .*

The Square-Root of BA , is the Semi-Axis of the Spheroid, with which, as Radius, describe the Circle $FALC$, then find the Points o, o , as in the last Problem, and so the Diameters OH ; Oh , as many as you please, one in the middle of every Inch of EB , will be sufficient: For the Area's of these Circles in Gallons, being added together, give the Content of the Part $OGHo$, and this doubled is the Content of the Cask $OHKO$.

But for our present purpose it will be sufficient to find the Square of a Diameter in the middle of each of the three first Inches, from B towards K , the Squares of the rest, and also the Area's of their

APPENDIX.

293

their Circles being easily obtained by their Common Difference. Thus,

From the Square of (O G) the Bung-Diameter, subtract the Square of (O H) the Head-Diameter, and divide the Square-Root of the Difference by (B E) the Semi-Length of the Cask. Then multiply this Quotient by .5, 1.5 and 2.5, and call the Products S.

Now, if from the Square of (O G) the Bung-Diameter, you subtract the Squares of S, the Remainders will be the Squares of the respective Diameters sought.

$$\begin{array}{rcl} \text{O G} = 32 & \text{Square is} = & 1024 \\ \text{O R} = 24 & \text{Square is} = & 576 \\ \hline & \text{Difference} = & 448 \end{array}$$

The Square Root of 448, is 21.16601, this divided by (B E) 20, gives 1.0583; which multiplied

$$\text{by } \left\{ \begin{array}{l} .5 \\ 1.5 \\ 2.5 \end{array} \right\} \text{ is } \left\{ \begin{array}{l} .52915 \\ 1.58745 \\ 2.64575 \end{array} \right\} \text{ squared is } \left\{ \begin{array}{l} .28 \\ 2.52 \\ 7.00 \end{array} \right\} = \text{to S.}$$

Now the Square of O G, viz. 1024.

$$\begin{array}{rcl} \text{Less the} & \left\{ \begin{array}{l} .28 \\ 2.52 \end{array} \right\} & \left\{ \begin{array}{l} 1023.72 \\ 1021.48 \end{array} \right\} \\ \text{Square} & & \text{is} \\ \text{of S, viz} & \left\{ \begin{array}{l} 7.00 \end{array} \right\} & \left\{ \begin{array}{l} 1017.00 \end{array} \right\} \end{array}$$

which are the Squares of the three first Diameters, one in the middle of every Inch from O G, towards O K, or O H; and these multiplied severally

rally by .0027851, gives the Area's of the respective Circles in Ale-Gallons.

$$\text{viz. } \left\{ \begin{array}{l} 2.85116 \\ 2.84492 \\ 2.83244 \end{array} \right\} \begin{array}{l} .00624 \\ .01248 \\ .01248 \end{array} \quad .00624$$

Now if the second Inch be subtracted from the first, the Difference is .00624, and the third Inch subtracted from the second, the Difference is .01248. then, from .01248, subtract .00624, the difference will be .00624, which is the same throughout, as in the following Table. In which observe, that the first Differences are made by a continual Addition of the second Difference, and the Area's in the second Column are obtain'd by subtracting the first of the first Differences from the first Area, and so on. These Area's added together are 48.724 Gallons, equal to the Content of the part OGKO, which doubled is 97.448 the Content of the Cask OHKO, which was required

A Table shewing the Content of every Inch of half the middle Frustum of the Spheroid, OOKG, Fig. VII.

	Content of every Inch of the Part OOK O	1. Differ.	2. Differ.
1	2.85116		
2	2.84492	.00624	.00624
3	2.83244	.01248	.00624
4	2.81372	.01872	.00624
5	2.78876	.02496	.00624
6	2.75756	.03120	.00624
7	2.72012	.03744	.00624
8	2.67644	.04368	.00624
9	2.62652	.04992	.00624
10	2.57036	.05616	.00624
11	2.50796	.06240	.00624
12	2.43932	.06864	.00624
13	2.36444	.07488	.00624
14	2.28332	.08112	.00624
15	2.19526	.08736	.00624
16	2.10236	.09360	.00624
17	2.00252	.09984	.00624
18	1.89644	1.0608	.00624
19	1.78412	1.1232	.00624
20	1.66556	1.1856	

Total = 48.724

The Double is 97.448 = Content of the Cask.

Of a Circular Spindle.

IF the Arch of a Circle $Z A Z$ (Fig. 8.) be turn'd about upon its Cord $Z B Z$, it will describe a Solid $Z O A O$, $Z F C E$, and this Solid, I call a *Circular Spindle*.

Problem IX. Figure 8.

To describe a Circular Spindle, the middle Frustums of which shall represent a Cask, whose Bung-Diameter $A C$, is 32 Inches, the Diameters at the Heads $O E$, and $Q F$ are each 24 Inches, and the Length $G H$, 40 Inches. And to find the Content of such Cask in Ale-Gallons.

DIvide the Square of the Semi-Length of the Cask, by half the Difference between the Bung and Head-Diameters, and to the Quotient add the said half Difference; the half of this Sum is equal to the Radius $A D$; with which (upon D as a Center) describe the Arch $L A M$, draw the Line $L M$, and make $A D$ perpendicular thereto. Make $A B$ equal to 16 the Semi-Diameter at the Bung. Draw the Line ZZ parallel to $L M$, and make $B C$, equal to $B A$. Through the Points $Z C Z$, describe the Arch $Z E C F Z$, set off the Semi-Length of the Cask from B , to G and H , through which Points, draw Lines parallel

APPENDIX.

297

parallel to AD . So shall OE and OF , be the Head-Diameters of the Cask, and each 24.

To find a Diameter in the middle of every Inch from AC , towards OE . Put $S = 60$. It is manifest, that the Square of OD , less the Square of S , is equal to the Square of Og . And if from the Square-Root of Og , you take Og , the Remainder Gg , will be the Semi-Diameter; which double gives the Diameter OE : And if we put the first $S = .5$, the second $= 1.5$ the third $= 2.5$, &c. and proceed as here directed, we shall have a Diameter in the middle of every Inch, as in the second Column of the following Table, against which, in the third Column, are the Area's in Ale-Gallons, that is, the Content of so many Circular Plains of one Inch thick, the Sum of these is 48.3938, the Content of the part $CEO A$; which doubled is 96.7876, the Content of the Cask required.

A Table shewing the Content of half the middle Frustum of the *Circular Spindle* OECFOA. Fig. VIII.

	Diam. in the middle of e- very Inch of B. G.	Area in Ale Gall.
1	31.9952	2.8510
2	31.9567	2.8442
3	31.8797	2.8303
4	31.7642	2.8090
5	31.6098	2.7818
6	31.4165	2.7491
7	31.1843	2.7084
8	30.9126	2.6614
9	30.6012	2.6309
10	30.2497	2.5464
11	29.8577	2.4828
12	29.4248	2.4113
13	28.9505	2.3342
14	28.4340	2.2517
15	27.8749	2.1640
16	27.2723	2.0615
17	26.6255	1.9744
18	25.9336	1.8733
19	25.1957	1.7680
20	24.4016	01.6580

Sum = 48.3938

Douhled is = 96.7876 = Content
of the Cask.

From what hath been said, it appears, that the middle Frustum of each of the four Solids last mention'd, represent a close Cask, (*viz.* a *Butt*, *Pipe*, *Hogshead* &c. And if a Cask whose Diameters and Length are as above supposed,

Viz. { Bung }
 { Head } Diameter } 24 } Inches.
 { Length } ———— 40 }

I say

I say, if such Cask be the middle Frustum of a *Spheroid* (as OHKO Fig. VII.) the Content (by Prob. VIII.) will be —————

Ale-Gall.

97.448

If the middle Frustum of a *Circular Spindle*. (as OEFO, Fig. VIII.) the Content (by Probl. IX.) will be —————

96.787

If the middle Frustum of a *Parabolick Spindle*, (as DCEF, Fig. V.) the Content by Probl. IV. will be —————

96.498

If the middle Frustum of a *Hyperbolick Spindle* (as OEFO, Fig. VI.) the Content by Problem. VI. will be —————

96.200

And if (as some Writers suppose) the said Cask be the middle Frustum of two *Parabolick Conoids*, abutting upon one common Base, (as ODOOBGO. Fig. III.) the Content by Probl. III. will be ————

89.122

If the middle Frustum of two *Cones* abutting upon one common Base, the Content will be ————

87.934

But neither of the two last do often (if at all) occur in the Practice of Gauging.

And though in several Books on this Subject we meet with the Terms *Frustum* of a *Spheroid*, *Conoid*, *Parabolick Conoid*, *Parabolick Spindle*, &c. the Writers of such Tracts have not (that I can find) told us what they mean by these Terms, nor will the Rules they have given find the Content of some of these Solids tolerably near the Truth.

For Instance,

If the Cask above-mentioned be the middle Frustum of a *Parabolick Spindle*, the Content (as we have shewn above) is 96.498, Ale-Gallons. Where-

Whereas Mr. *Jordain*, in his *Duodecimal Arithmetick*, makes it but 93.877-

Mr. *Dary*, at the end of his *Miscellanies*, makes the Content but 90.948.

And Mr. *Hunt*, in his *Practical Gauging Epitomized*, makes the Content of this Cask but 87.930, which is above 8 Gallons less than the true Content.

If what has been said in this Appendix shall give any Light to the Practical Gauger, in conceiving the Nature and Properties of the Solids here mentioned, it is as much as was at first designed.

For although the middle Frustum of each of the four Solids described in Prob. IV, VI, VIII, and IX. do very properly represent a Cask, and the Contents there found be sufficiently exact; yet the Contents of three of 'em differ so little one from another, that if every Cask were in the Form of one of those three Solids, it would be needless to make any Distinction, because upon sight of a Cask, one could not tell to which of those Solids it agreed.

And there being very few (if any) close Casks of such Diameters and Lengths as we have frequently mention'd, which contain more than the middle Frustum of a *Spheroid* (this having the greatest Curvature between the Head and Bung;) but very many Casks (of such Diameters and Length) which will not contain so much as the middle Frustum of a *Parabolick Hyperbolick Spindle*, (their Staves between the Head and Bung, being nearer a straight Line than either of these Frustums;) I must refer the Practical Gauger to the Rules laid down in the four first Problems of the Ninth Section of the foregoing Treatise, by which the Content of any Cask may be readily found (either by the Pen or Instrument) sufficiently near the Truth.

F I N I S.

20 FEB 69

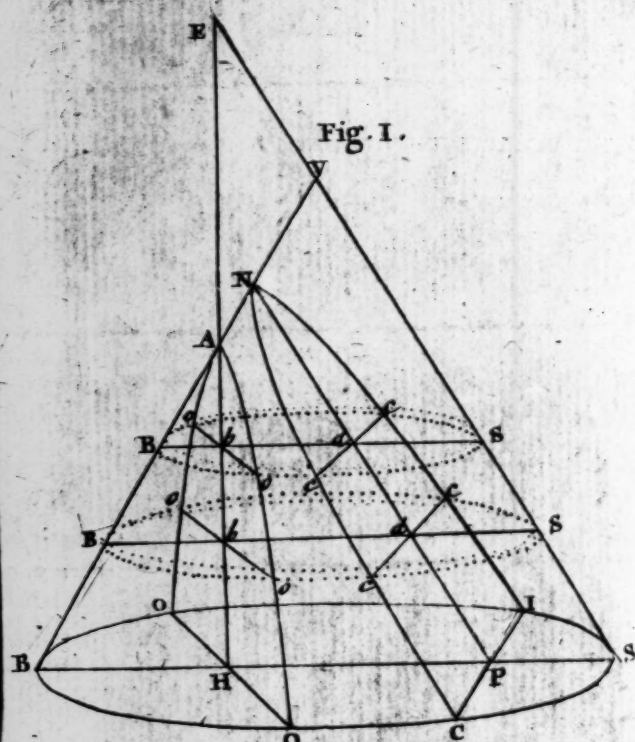


Fig. I.

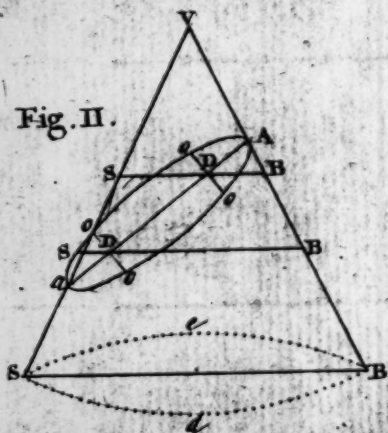


Fig. II.

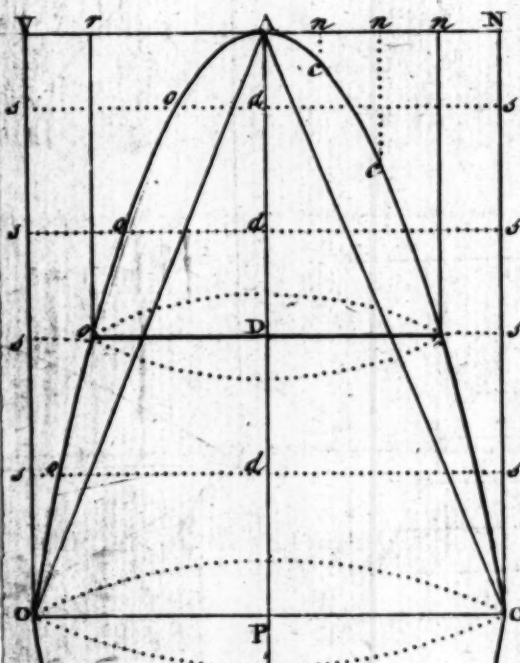


Fig. III.

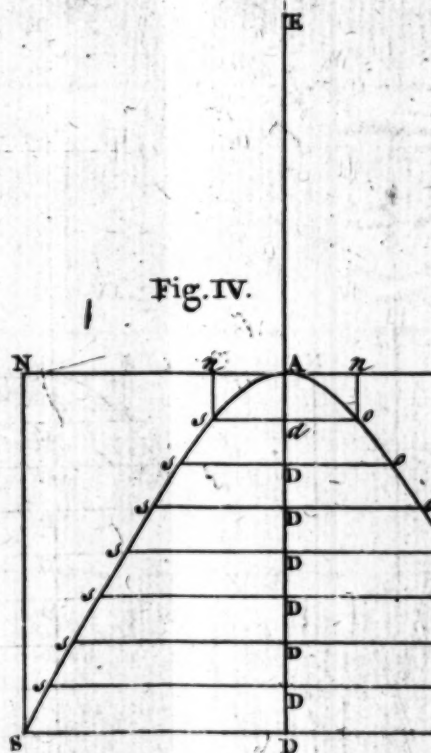
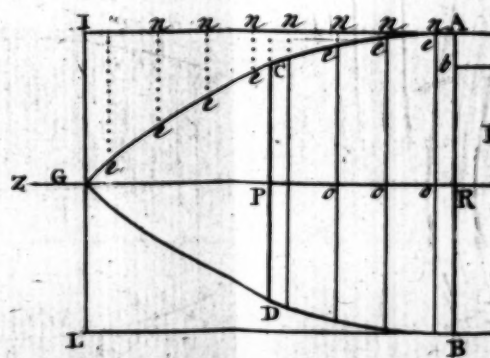


Fig. IV.



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